Mix design programming for normal concrete using cubic equation

Eugene Yudhistira Baggio, Toni Hartono Bagio, Julistyana Tistogondo
Civil Engineering Department Narotama University Surabaya, INDONESIA
E-mail: tony@narotama.ac.id, eugenebaggio@gmail.com

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ABSTRACT

A computer programming requires inputs and processes to produce output, good data processing requires a formula, in programming when using tables is very inefficient, so an approach is needed in this case the existing table tables are converted into formulas, making it easier to process programming. Mix Design for normal concrete, in general, always use tables that have SNI (ACI), the tables are modified with Polynomial Regression to be equations, tables in SNI (ACI) that are transformed into equations. The equation produced, using polynomial regression, with a value of $R^2 \approx 1$.

The equation to find weight of mixing water ($W_w$) = $y_{1j}$, $y_{2j}$, with main variable aggregate maximum, additional variables are slump, the second equation is volume air content (VAC) = $y_{3i}$, additional variables are exposures, water cement ratio (WCR) = $y_{4i}$, with aggregate maximum variables, while additional variables are air requirements in concrete (non-air entrained/air entrained), weight of coarse aggregate (WCA) = $y_{5i}$, with two variables aggregate maximum and FM (Finest Modulus), and initial estimates of fresh concrete weight (WFC) = $y_{6i}$

Keywords: programming; cubic equation; polynomial regression; mix design; aggregate maximum.

INTRODUCTION

In general, according to [1], [2], concrete consists of water, cement, pozzolanic, coarse aggregates, fine aggregates, besides that concrete may contain air cavities, [3], [4], [5], it can also be intentionally given air, both air cavities are calculated using a mixture composition between water cement, pozzolanic, coarse aggregates, fine aggregates, [6], [7], [8] with the limitation of making normal concrete mixes [7], [9], [10], [11], which is not high-quality concrete [12], the variable used is not an exact thing, so the difference between designers is very reasonable, so the table tables in SNI (ACI), to facilitate calculations, are replaced by equation [13], [14], [15]. When on a computer program [16], [17], it is very inefficient to use a table, so it is necessary to change the existing table, into a Polynomial equation [18],[19]. Final result comparation between equation’s method vs table’s method [20]

RESEARCH METHODS

ACI [3], table A1.5.3.3 or SNI [5] table 2 convert to cubic equation [18]. Non-air-entrained concrete:

$\text{Slump} = 25-50$ (R²=0.9977)

$y_{11} = -0.0000111^*x^3 + 0.0089425^*x^2 - 1.8147174^*x + 221.533752$ (1.a)

$\text{Slump} = 50-75$ (R²=0.9957)

$y_{12} = -0.0000291^*x^3 + 0.0131584^*x^2 - 2.088542^*x + 234.3143$ (1.b)

$\text{Slump} = 75-100$ (R²=0.9940)

$y_{13} = -0.0000426^*x^3 + 0.0162374^*x^2 - 2.2785993^*x + 244.2855808$ (1.c)

$\text{Slump} = 100-125$ (R²=0.9973)

$y_{14} = -0.0000488^*x^3 + 0.0162374^*x^2 - 2.2785993^*x + 244.2855808$ (1.d)

$\text{Slump} = 150-175$ (R²=0.9965)

$y_{15} = -0.00004987^*x^3 + 0.0728043^*x^2 - 4.272658^*x + 267.7511$ (1.e)

$\text{Slump} = 150-175$ (R²=0.9959)

$y_{16} = -0.000536^*x^3 + 0.0827147^*x^2 - 4.7825228^*x + 279.4357583$ (1.f)

(see figure 1)

Air-entrained concrete.
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\[
\text{Slump} = 25-50 \quad (R^2=0.9968) \\
y21 = -0.0000038x^3 + 0.0054866x^2 - 1.2983588x + 191.1850543 \\
\text{Slump} = 50-75 \quad (R^2=0.9934) \\
y22 = -0.0000055x^3 + 0.0064939x^2 - 1.453547x + 203.4307 \\
\text{Slump} = 75-100 \quad (R^2=0.9910) \\
y23 = -0.0000097x^3 + 0.0076815x^2 - 1.558815x + 212.70388 \\
\text{Slump} = 100-125 \quad (R^2=0.9983) \\
y24 = -0.0004524x^3 + 0.0638114x^2 - 3.538278x + 236.2557 \\
\text{Slump} = 125-150 \quad (R^2=0.9972) \\
y25 = -0.0004390x^3 + 0.0649920x^2 - 3.690117x + 242.3494 \\
\text{Slump} = 150-175 \quad (R^2=0.9950) \\
y26 = -0.0003385x^3 + 0.0562362x^2 - 3.5060661x + 243.4023641
\]

**(2.a)**

\[
y21 = -0.0000038x^3 + 0.0054866x^2 - 1.2983588x + 191.1850543
\]

**(2.b)**

\[
y22 = -0.0000055x^3 + 0.0064939x^2 - 1.453547x + 203.4307
\]

**(2.c)**

\[
y23 = -0.0000097x^3 + 0.0076815x^2 - 1.558815x + 212.70388
\]

**(2.d)**

\[
y24 = -0.0004524x^3 + 0.0638114x^2 - 3.538278x + 236.2557
\]

**(2.e)**

\[
y25 = -0.0004390x^3 + 0.0649920x^2 - 3.690117x + 242.3494
\]

**(2.f)**

\[
y26 = -0.0003385x^3 + 0.0562362x^2 - 3.5060661x + 243.4023641
\]

**(see figure 2)**

where:

\[
x = \text{Aggregate Max (mm)}; \\
y_{ij} = \text{mixing water (kg) note of } i, j \\
i = 1 \text{ (non-air-entrained concrete)} \\
2 = \text{ (air-entrained concrete)} \\
\text{slump’s number (1, 2, 3, 4, 5, 6)}
\]

Level of exposure : 0) Trapped air (%)
\[
y30 = -0.0000052x^3 + 0.0014847x^2 - 0.1316092x + 4.0214712 \quad (R^2=0.9958)
\]

Level of exposure : 1) Mild (%)
\[
y31 = -0.0000041x^3 + 0.0012139x^2 - 0.1196683x + 5.4068884 \quad (R^2=0.9955)
\]

Level of exposure : 2) Moderate (%)
\[
y32 = -0.0000034x^3 + 0.0009770x^2 - 0.0943738x + 6.5976472 \quad (R^2=0.9668)
\]

Level of exposure : 3) Extreme (%)
\[
y33 = -0.0000046x^3 + 0.0013253x^2 - 0.1233447x + 8.3344736 \quad (R^2=0.9712)
\]

**(see figure 3)**

**Figure 1.** Aggregate maximum vs weight of water (Non-air-entrained)
Figure 2. Aggregate maximum vs weight of water (Air-entrained)

Figure 3. Aggregate maximum vs air content

where:

\[ x = \text{Aggregate Max (mm)} \]
\[ y_{3i} = \text{air content (\%)} \]

note of \( i \)

- non-air-entrained concrete, \( i = 0 \)
- air-entrained-concrete,
  - if level of exposure = “mild”, \( i = 1 \)
  - if level of exposure = “moderate”, \( i = 2 \)
  - if level of exposure = “extreme”, \( i = 3 \)

ACI[3], table A.1.5.3.4(a) or SNI [5] table 3 convert to cubic equation

- Non-air-entrained concrete

\[ y_{41} = -0.00000022x^3 + 0.0003905x^2 - 0.0308968x + 1.1721429 \] (\( R^2=0.9996 \)) \hspace{1cm} (4.a)

- Air-entrained concrete

\[ y_{42} = -0.00000067x^3 + 0.0007571x^2 - 0.0401905x + 1.1548571 \] (\( R^2=0.9999 \)) \hspace{1cm} (4.b)

(see figure 4)
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Figure 4. fck vs w/c ratio

where:
\[ x = fck \text{ (MPa)} \]
\[ y_{i} = \text{water cement ratio (unit less)} \]
notes of \( i \):
\[ i = 1 \text{ (Non-air-entrained)} \]
\[ i = 2 \text{ (Air-entrained)} \]

ACI[3], table A.1.5.3.4(b) or SNI [5] table 4 is condition of water content ratio maximum,
IF Exposures = “Mild” THEN WcMax = 1
IF Exposures = “Severe” AND Type = “thin” AND Structure = “wet” THEN WcMax = 0.45
IF Exposures = “Severe” AND Type = “thin” AND Structure = “sea” THEN WcMax = 0.40
IF Exposures = “Severe” AND Type = “other” AND Structure = “wet” THEN WcMax = 0.50
IF Exposures = “Severe” AND Type = “other” AND Structure = “sea” THEN WcMax = 0.45

ACI[3], table A.1.5.3.6 or SNI [5] table 5 convert to cubic equation:
9.5 mm ≤ x < 25 mm (R²=1)
\[ y_{51} = + 0.000118027*x^3 - 0.006863402*x^2 + 0.1378855*x - 0.051683 - 0.1(z) \] (5.a)
25 mm ≤ x ≤ 150 mm (R²=0.9999)
\[ y_{52} = + 0.000000121*x^3 - 0.000042577*x^2 + 0.0054684*x + 0.8381061 - 0.1(z) \] (5.b)
where:
\[ x = \text{aggregate max (mm)} \]
\[ z = \text{FM (Finest Modulus) of Fine Aggregate (unit less)} \]
\[ y_{5i} = \text{volume of coarse aggregate (m³)} \]
notes of \( i \):
i = 1, if \( x < 25\text{ mm} \); i = 2, if \( x \geq 25\text{ mm} \)
(see figure 5)

ACI[3], table A.1.5.3.7.1 or SNI [5] table 6 convert to cubic equation:
Non-air-entrained concrete
\[ y_{61} = + 0.000214443*x^3 - 0.06765698*x^2 + 7.360915*x + 2224.583 \quad (R^2 = 0.9945) \] (6.a)
Air-entrained concrete
\[ y_{62} = + 0.000128711*x^3 - 0.04674232*x^2 + 5.941731*x + 2161.381 \quad (R^2 = 0.9836) \] (6.b)
(see figure 6)
where:
\[ x = \text{Aggregate Max (mm)} \]
\[ y_{6i} = \text{mass of fresh concrete (kg/m³)} \]
notes of \( i \):
i = 1, (non-air-entrained),
i = 2, (air-entrained)
\[ F_W = \frac{p}{c+p} \] ................................................................. (7)

\[ F_V = \frac{F_W}{F_W + \frac{G_p}{G_c} (1-F_W)} \] ................................................................. (8)

Fw = pozzolanic materials percentage by weight (expression in decimal)
Fv = pozzolanic materials percentage by volume (expression in decimal)
p = weight of pozzolanic materials
c = weight of cement
Gp = specific gravity of pozzolanic
Gc = specific gravity of cement

Method of concrete mix design is as follows:

Data requirements:
1. Average strength, \( f_{ck} \) (MPa)
2. Choose Slump [ 1, 2, 3, 4, 5, 6 ]
   (1="25-50"; 2="50-75"; 3="75-100"; 4="100-125"; 5="125-150"; 6="150-175")
3. Pozzolanic
   a. \( F_W \) pozzolanic (decimal)
   b. \( G_p \) (specific gravity of pozzolanic)
4. Cement
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5. Exposure of Concrete
   a. Choose Exposure = [1, 2]  
      (1 = Mild; 2 = Severe)
   b. If Exposure = 1("Mild"), go to step 6
   c. Choose type of structure, Type = [1, 2]  
      (1 = Thins; 2 = Others)
   d. Choose structure, Structure = [1, 2]  
      (1 = Wet; 2 = Sea or Sulphates)

6. Fine aggregate:
   a. FM (Fineness Modulus)
   b. Gfa (specific gravity of fine aggregate)
   c. AbFA (water absorption of fine aggregate, %)
   d. MoFA (moisture of fine aggregate, %)

7. Coarse aggregate
   a. DRM (Dry-rodded mass)
   b. Gca (specific gravity of coarse aggregate)
   c. AbCA (water absorption of coarse aggregate, %)
   d. MoCA (moisture of coarse aggregates, %)
   e. AggMax (maximum size of aggregate, mm)

Calculation:
8. Calculate mixing water (WW)
   WW = yij
   where:
   i = Entrap [1, 2]
   j = Slump [1, 2, 3, 4, 5, 6]
9. Water cement ratio
   a. Calculate water cement ratio (WCR)
      WCR = y4i; i = Entrap [1, 2]
   b. Calculate water cement ratio maximum (WMX)
      IF Exposure = 1 THEN WMX = 1
      IF Exposure = 2 AND Type = 1 AND Structure = 1 THEN WMX = 0.45;
      IF Exposure = 2 AND Type = 1 AND Structure = 2 THEN WMX = 0.40;
      IF Exposure = 2 AND Type = 2 AND Structure = 1 THEN WMX = 0.50;
      IF Exposure = 2 AND Type = 2 AND Structure = 2 THEN WMX = 0.45;
   c. Calculate final water cement ratio (WFR)
      WFR = MIN (WCR, WMX)
   d. Calculate final water cement ratio, by volume (WFRv)
      Fv = Fw/[Fw+Gp/Gc*(1-Fw)]
      WFRv = Gc * WFR / [Gc * (1 - Fv) + Gp * Fv]
      (Fw, Fv = expression in decimal)

Mass bases

Final method of concrete mix design base on mass is as follows:
10. Calculate Weight of Cement (WC)
    WC = Fw * WW/WFR
11. Calculate Weight of Pozzolanic (WP)
    WP = (1 – Fw) * WW/WFR
12. Calculate Weight of Coarse Aggregate (WCA)
    WCA = y5i * DRM;
        where: IF AggMax < 25 mm THEN i = 1 ELSE i = 2
13. Calculate Weight of Fresh concrete (WFC)
14. Calculate Weight of Fine Aggregate (WFA)
\[ WFA = WFC - (WW + WC + WP + WCA) \]

15. Adjusted weight of mixing water (WAW)
\[ WAW = WW - WCA(\text{MoCA} - \text{AbCA})/100 - WFA(\text{MoFA} - \text{AbFA})/100 \]

16. Adjusted weight of fine aggregate (WAF)
\[ WAF = WFA*(1 + \text{MoFA}/100) \]

17. Adjusted weight of coarse aggregate (WAC)
\[ WAC = WCA*(1 + \text{MoCA}/100) \]

18. Final result (WAW, WC, WP, WAF, WAC)

**Volume bases**

Final method of concrete mix design base on volume are follows:

10. Calculate Volume of mixing water (VW)
\[ VW = WW/1000 \]

11. Calculate Volume of Cement (VC)
\[ VC = (1 - Fv) * WW/(1000*Gc*WFRv) \]

12. Calculate Volume of Pozzolanic (VP)
\[ VP = Fv * WW/(1000*Gp*WFRv) \]

13. Calculate Volume of Coarse Aggregate (VCA)
\[ VCA = y5i * DRM / (1000*Gca) \]
where: IF AggMax < 25 mm THEN \( i = 1 \) ELSE \( i = 2 \)

14. Calculate Volume of Air Content (VAC)
\[ VAC = y3i / 100 \]
where: \( i = \text{level} \ [0, 1, 2, 3] \)

15. Calculate Volume of Fine Aggregate (VFA)
\[ VFA = 1 - (VW + VC + VP + VCA + VAC) \]

16. Calculate Weight of mixing water (WWv)
\[ WWv = VW*1000 \]

17. Calculate Weight of Cement (WCv)
\[ WCv = Vc * 1000 * Gc \]

18. Calculate Weight of Pozzolanic (WPv)
\[ WPv = Vp * 1000 * Gp \]

19. Calculate Weight of Coarse Aggregate (WCAv)
\[ WCAv = VCA * 1000 * Gca \]

20. Calculate Weight of Fine Aggregate (WFAv)
\[ WFAv = VFA * 1000 * Gfa \]

21. Adjusted weight of mixing water (WAWv)
\[ WAWv = WWv - WCAv*(\text{MoCA} - \text{AbCA})/100 - FAv*(\text{MoFA} - \text{AbFA})/100 \]

22. Adjusted weight of fine aggregate (WAFv)
\[ WAFv = WFAv*(1 + \text{MoFA}/100) \]

23. Adjusted weight of coarse aggregate (WACv)
\[ WACv = WCAv*(1 + \text{MoCA}/100) \]

24. Final result (WAWv, WCv, WPv, WAFv, WACv)

**RESULTS AND DISCUSSION**

Required average strength (fck) will be 24 MPa with slump of 75 to 100 mm ( = 3). Cement : Type I with specific gravity (Gc) = 3.15. Concrete Air-entrapped (Entrap) = Non-air-entrapped ( = 1), Level air content (Level) = Non-air ( = 0), Exposure = Mild ( = 1), type = Null, structure = Null. Coarse aggregate : bulk specific gravity (Gca) = 2.68, absorption (AbCA) = 0.5 % and moisture content (MoCA) = 3%, has a nominal maximum size (AggMax) = 37.5 mm and dry-rodded mass (DRM) = 1600 kg/m³. Fine aggregate : bulk specific gravity (Gfa) = 2.64, absorption (AbFA) = 0.7 %, moisture content (MoFA) = 5% , fineness modulus (FM) = 2.8. Fly ash : specific gravity (Gp) = 2.14 and pozzolanic prosentase by weight (Fw) = 15%
Table 1. Result using cubic equation

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg)</th>
<th>Volume (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>115</td>
<td>117</td>
</tr>
<tr>
<td>Fly Ash</td>
<td>43</td>
<td>50</td>
</tr>
<tr>
<td>Cement</td>
<td>243</td>
<td>224</td>
</tr>
<tr>
<td>Coarse Aggregate (wet)</td>
<td>1169</td>
<td>1170</td>
</tr>
<tr>
<td>Fine Aggregate (wet)</td>
<td>858</td>
<td>820</td>
</tr>
<tr>
<td>Total</td>
<td>2428</td>
<td>2381</td>
</tr>
</tbody>
</table>

Table 2. Result using direct table

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg)</th>
<th>Volume (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>118</td>
<td>119</td>
</tr>
<tr>
<td>Fly Ash</td>
<td>43</td>
<td>52</td>
</tr>
<tr>
<td>Cement</td>
<td>246</td>
<td>224</td>
</tr>
<tr>
<td>Coarse Aggregate (wet)</td>
<td>1170</td>
<td>1170</td>
</tr>
<tr>
<td>Fine Aggregate (wet)</td>
<td>844</td>
<td>810</td>
</tr>
<tr>
<td>Total</td>
<td>2421</td>
<td>2375</td>
</tr>
</tbody>
</table>

Table 3. Comparison Equation method vs Table method base on mass

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>115</td>
<td>118</td>
<td>3</td>
<td>2.61%</td>
<td>2.54%</td>
<td></td>
</tr>
<tr>
<td>Fly Ash</td>
<td>43</td>
<td>43</td>
<td>0</td>
<td>0.00%</td>
<td>0.00%</td>
<td></td>
</tr>
<tr>
<td>Cement</td>
<td>243</td>
<td>246</td>
<td>3</td>
<td>1.23%</td>
<td>1.22%</td>
<td></td>
</tr>
<tr>
<td>Coarse Aggregate (wet)</td>
<td>1169</td>
<td>1170</td>
<td>1</td>
<td>0.09%</td>
<td>0.09%</td>
<td></td>
</tr>
<tr>
<td>Fine Aggregate (wet)</td>
<td>858</td>
<td>844</td>
<td>14</td>
<td>1.63%</td>
<td>1.66%</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2428</td>
<td>2421</td>
<td>-</td>
<td>0.29%</td>
<td>0.29%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Comparison Equation method vs Table method base on absolute volume

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>117</td>
<td>119</td>
<td>2</td>
<td>1.71%</td>
<td>1.68%</td>
<td></td>
</tr>
<tr>
<td>Fly Ash</td>
<td>50</td>
<td>52</td>
<td>2</td>
<td>4.00%</td>
<td>3.85%</td>
<td></td>
</tr>
<tr>
<td>Cement</td>
<td>224</td>
<td>224</td>
<td>0</td>
<td>0.00%</td>
<td>0.00%</td>
<td></td>
</tr>
<tr>
<td>Coarse Aggregate (wet)</td>
<td>1170</td>
<td>1170</td>
<td>0</td>
<td>0.00%</td>
<td>0.00%</td>
<td></td>
</tr>
<tr>
<td>Fine Aggregate (wet)</td>
<td>820</td>
<td>810</td>
<td>10</td>
<td>1.22%</td>
<td>1.23%</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2381</td>
<td>2375</td>
<td>-</td>
<td>0.25%</td>
<td>0.25%</td>
<td></td>
</tr>
</tbody>
</table>

Note: Table 3 and Table 4


CONCLUSION

Comparison between Equation’s method and Table’s method base on Mass. Water 2.61% and 2.54%, Fly Ash 0% and 0%, Cement 1.23% and 1.22%, Coarse Aggregate 0.09% and 0.09%, Fine Aggregate 1.63% and 1.66%, total 0.29% and 0.29%, all result less than 5%. Equation Method can be used for Mass. Comparison between Equation’s method and Table’s method base on Volume. Water 1.71% and 1.68%, Fly Ash 4% and 3.85%, Cement and Coarse Aggregate 0% and 0%, Fine Aggregate 1.22% and 1.23%, total 0.25%, all result less than 5%. Equation Method can be used for Volume Base.

Calculation of mixing water (Non-air-entrained) used equation (1.a) …. (1.f).
Calculation of mixing water (Air-entrained) used equation (2.a) …. (2.f).
Calculation of air content used equation (3.a) …. (3.d).
Calculation of water content ratio used equation (4.a) or (4.b).
Calculation of Volume Coarse Aggregate used equation (5.a) or (5.b).
Calculation of fresh concrete used equation (6.a) or (6.b).

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