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Crop Insurance Model Based on Maximum Daily Rainfall and Maximum Daily Temperature Index

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ABSTRACT

A loss insurance model of risk for agricultural commodities that considers maximum daily rainfall and maximum daily temperature is introduced in this paper. This model requires bivariate distribution of maximum daily rainfall and maximum daily temperature in a specific region. Characteristics of particular agricultural commodity is also needed in the region where the two variables are being insured. The bivariate distribution and commodity characteristics are combined to obtain exit. exit is a benchmark value that causes the total crop failure and gives full benefit claim to policyholder. The case study was demonstrated by using data on maximum daily rainfall and temperature in Dramaga Bogor from September to December during 38 years (1984-2021). Data was collected from Jawa Barat Climatology Station. Frank Copula is better to represent the bivariate distribution of data. Furthermore, two scenarios had given the premiums as IDR 2 482 925 per hectare and IDR 1 495 660 per hectare. This crop insurance model based on maximum daily rainfall and maximum daily temperature index could be the basis for the next developing of crop insurance model.

Keywords: crop insurance model; exit; maximum daily rainfall distribution; maximum daily temperature distribution; premium.

INTRODUCTION

Total crop failure in the agricultural sector cannot be separated from uncertain factors such as rainfall and temperature. Rainfall is the height of rainwater that collects in a flat place, does not evaporate, does not seep and does not flow. Based on information obtained from the Meteorology, Climatology and Geophysical Agency (BMKG), one millimeter of rainfall is equal to one liter of rainwater in an area of one square meter (1 mm = 1 liter/m^2). Rainfall is recorded by the rainfall station continuously (BMKG, 2022). Meanwhile, temperature is said to be a degree of hot or cold (Munawir et al, 2022), which is measured based on a certain scale using measuring instrument (Munawir, 2017).

An alternative tool to insure the risk of agricultural loss which can specifically be used by farmers in protecting agricultural production in the event of total crop failure due to uncertainty is called by crop insurance based on weather index. One of the developments of the crop insurance model based on weather index is crop insurance model based on rainfall index. The compensation for loss takes into account the realization of rainfall measured during the insurance period at certain climatology station (Boer, 2012; Muna, 2017; Muna, 2019a). The design of the weather index insurance can also take into account several variables such as daily rainfall, maximum and minimum daily temperature, and daily duration of irradiation. (Boyd, et al., 2020)

In Muna (2019b), crop insurance model based on rainfall index that had been developed built an index by rainfall distribution and characteristics of agricultural commodity. Several univariate distributions are Normal distribution and Generalized Extreme Value (GEV) distribution. On the other hand, Copula is the main tool that is widely used by researchers for multivariate intensity analysis (Zhang, et al., 2006). Copula is a method that can be used to model the relationship between two or more variables without assuming the marginal distribution of these variables. Copula uses two or more random variables with the marginal distribution of each random variable being different or even the marginal distribution is unknown (Cherubini & Luciano, 2004).

The benefit claim model for the crop insurance model based on weather index requires parameter from the characteristics of agricultural commodity and the probability distribution to divide several payment conditions such as full payment, partial payment, and no payment. This claim model can further calculate the premium that policyholders have to pay for protection. The premium is the multiplication between the amount of the benefit claim and its probability (Muna, 2019b).

This paper aims to develop crop insurance model based on maximum daily rainfall and maximum daily temperature index. The benefit claim model is made into two payment conditions which are full payment and no payment. In addition, the best marginal distribution model is chosen for the two variables between the Normal distribution model and GEV distribution model. Then, the bivariate distribution model is approached with Copula. Finally, the formula and calculation of premium are gained for this crop model. The application used is *R i386 4.1.3* and Minitab 21.1 (64-bit).

Crop insurance model based on maximum daily rainfall and maximum daily temperature is developed by focused on literatures that are used. Literature are consisted of correlation analysis and probability theory are given as follows

Probability Theory
Random Variable

Defined $(\Omega, \mathcal{F}, P)$ as a probability space (*probability triple*). A measurable function from the set of all possible outcomes Ω to a measurable space is A called a random variable $X : \Omega \to$ A. X real value in general. Probability of X can be formulated as follows

$$P(X \in S) = P(\{\omega \in \Omega \mid X(\omega) \in S\})$$

where P is the probability measure which is completed by Ω and $S \subseteq$ A (McKay, 2019)

Cumulative Distribution Function

Suppose X is a random variable. The cumulative distribution function is written as $F_X(x) = P(X \leq x)$ (Hogg, et al., 2014)

Continuous Random Variable

A random variable is called a continuous random variable if the cumulative distribution function $F_X(x)$ is a continuous function for each $x \in \mathbb{R}$. For continuous random variable $X,$ there is no discrete point mass. For example, if it is X continuous then $P(X = x) = 0$ for every $x \in \mathbb{R}$. A continuous random variable is completely continuous,

$$F_X(x) = \int_{-\infty}^{x} f_X(u)\, du,$$

for some functions $f_X(x)$. The function $f_X(x)$is called the probability density function of X.

(Hogg, et al., 2014)

Expected Value of Continuous Distribution

If Xit is a continuous random variable with probability density function $f_X(x)$ then the expected value of $X$is

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx,$$

provided that the integral is absolutely convergent (Hogg, *et al.*, 2014).

Joint Cumulative Distribution Function

Given a random experiment with a sample space C, consider two random variables X_1 and X_2, where $X_1(c) = x_1, X_2(c) = x_2,$ then (X_1, X_2) is a vector of variables. The space of (X_1, X_2) is an ordered set $D = \{(x_1, x_2): x_1 = X_1(c), x_2 = X_2(c), \mathrm{c} \in \mathrm{C}\ \}$. The joint cumulative distribution function of (X_1, X_2) is as follows:

$$F_{X_1, X_2}(x_1, x_2) = Pr(\{X_1 \leq x_1\} \cap \{X_2 \leq x_2\})$$

For all$(x_1, x_2) \in \mathbb{R}^2$. (Hogg, et. al, 2019)

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Marginal Distribution

Let (X_1, X_2) be a vector of variables then X_1 and X_2 are random variables. The distribution of each X_1 and X_2 can be obtained from the joint cumulative distribution function (X_1, X_2).

$$\{X_1 \leq x_1\} = \{X_1 \leq x_1\} \cap \{-\infty < X_2 < \infty\} = \{ X_1 \leq x_1, -\infty < X_2 < \infty\}$$

So the marginal distribution $F_{X_1}(x_1) = Pr(X_1 \leq x_1, -\infty < X_2 < \infty)$ (Hogg, et. al, 2019).

Distribution Model for One Variable (Univariate) and Two Variable (Bivariate)

The distribution models for one variable given in this paper are Normal distribution and *Generalized Extreme Value* (GEV) distribution. All models are used to model the maximum daily rainfall and maximum daily temperature. In addition, the distribution model of two variables as a joint cumulative distribution function used is the Copula model. Normal, GEV, and Copula distribution models are given as follows

Normal Model

Normal distribution or Gaussian, is a simple distribution, mathematically can be calculated using the average and standard deviation. Normal distribution processes a large number of random processes with fixed behavior. The sum of all these random components creates a random variable that converges on a normal distribution with the data distribution causing little effect (Lyon, 2014).

The normal distribution is a symmetrical distribution. A random variable (M) said to be normally distributed (μ, σ) if it has the following probability density function:

$$f_M(m) = \frac{1}{(2\,\pi)^{1/2}\sigma} \exp\left(-\frac{(m\; - \;\mu)^2}{2\,\sigma^2}\right), \tag{1}$$

for $m \in \mathbb{R}, \mu \in \mathbb{R}, \sigma > 0$. In addition, the cumulative distribution function of the Normal distribution is

$$F_M(m) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{m - \mu}{\sigma\, 2^{1/2}}\right)\right], \tag{2}$$

with the *error function* as follows ut

$$\operatorname{erf}(m) = \frac{2}{\pi} \int_0^m \exp(-s^2)\, ds. \tag{3}$$

Location (μ) states the average of data and the *scale* (σ) shows the deviation of data. If *location* (μ) and *scale* $(\sigma)$is known, the entire Normal distribution curve is known. The properties of the Normal distribution curve are illustrated by the following Figure 1. (Muna, 2017)

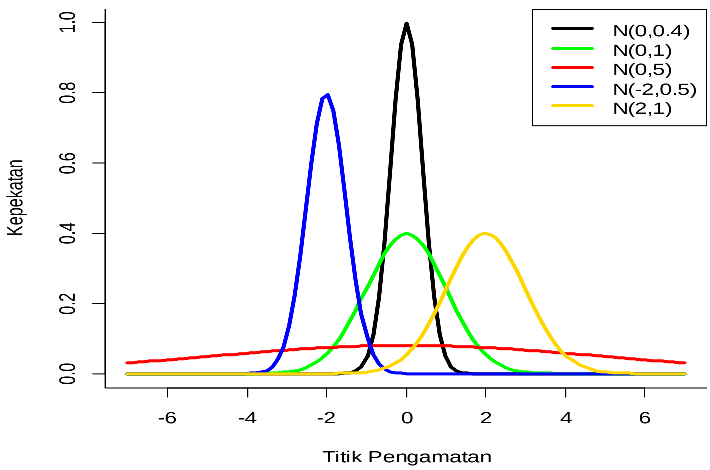

Figure 1. Probability density function illustration of Normal distribution (μ,σ) Source: (Muna, 2017)

Generalized Extreme Value (GEV) Model

GEV distribution introduced by Jenkinson in 1955, is a combination of three types of distribution families for extreme values, namely the Gumbel, Frechet, and *reversed* Weibull distributions derived by Fisher and Tippett into a single form (Hosking, et al., 1985). Suppose that $M_1, M_2, \dots, M_n$ the random variable which is independent, identical, and distributed GEV , then the extreme value variable converges on the cumulative distribution as follows

$$\text{I} : F_M(m) = \exp\left\{-\left(1+\xi\frac{m-\alpha}{\lambda}\right)^{-\frac{1}{\xi}}\right\}, \text{for } \xi \neq 0, \text{ and} \tag{4}$$

$$\text{II} : F_M(m) = \exp\left\{-\exp\left(-\frac{m-\alpha}{\lambda}\right)\right\}, \text{ for } \xi = 0.$$
(5)

where is M defined for $1+\xi\frac{m-\alpha}{\lambda} > 0, -\infty < \alpha < \infty,\ \lambda > 0$,and $-\infty < \xi < \infty$. α is the location parameter, λ is the scale parameter, and ξ is the shape parameter. (Muna, 2019b)

The shape parameter ξ determines the characteristics of the tip of the distribution. If $\xi < 0$ then the probability function has a finite right end point and if $\xi \geq 0$ then the probability function will have an infinite right end point . The parametric form of the GEV will lead to the Gumbel cumulative distribution for the limit $\xi \to 0$, the Frechet cumulative distribution if $\xi > 0$ with a heavy-tailed base distribution and the *reversed* Weibull cumulative distribution if $\xi < 0$ (Cai & Hames 2010).

The probability density function of the GEV distribution (α, λ, ξ) can be derived from the cumulative distribution function in Equations (4) and (5) . The formula for the probability density function of GEV is as follows

$$\text{I} : f_R(r) = \frac{1}{\lambda}\left(1+\xi\frac{r-\alpha}{\lambda}\right)^{-\frac{1}{\xi}-1} \exp\left[-\left(1+\xi\frac{r-\alpha}{\lambda}\right)^{-\frac{1}{\xi}}\right], \text{for } \xi \neq 0, \text{ and} \tag{6}$$

$$\text{II} : f_R(r) = \frac{1}{\lambda}\exp\left(-\frac{r-\alpha}{\lambda}\right) \exp\left[-\exp\left(-\frac{r-\alpha}{\lambda}\right)\right], \text{for } \xi = 0, \tag{7}$$

with parameters $\alpha, \lambda, \xi \in \mathbb{R}$. (Muna, 2019b)

Some of the probability density curves of the GEV distribution can be seen in Figure 2.

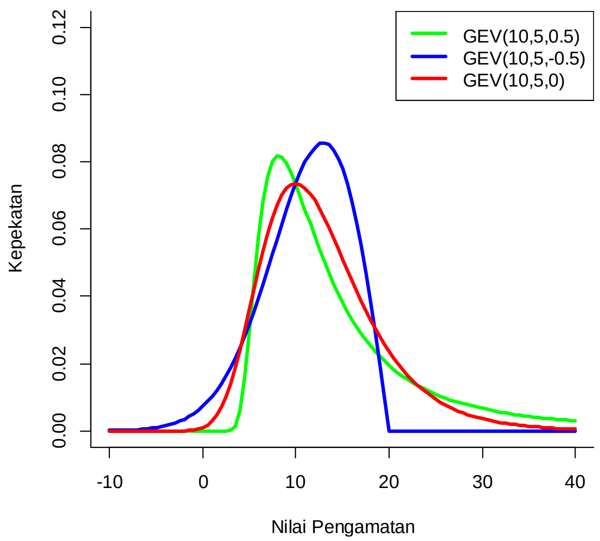

Figure 2. Probability density function illustration of GEV distribution (α,λ ,ξ) Source: (Muna, 2019)

Copula

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Bayu (2022) states that Copula is a function that combines several distributions of marginal data into a common distribution (Cherubini, et al., 2004). Suppose a random vector $(X_1, X_2) \in \mathbb{R}$ has a continuous marginal distribution vector X, that is, the marginal cumulative distribution function. $F_i(x) = Pr(X_1 \le x)$ is a continuous function that does not descend with $F_i(-\infty) = 0$ and $F_i(-\infty) = 1.$ For the application of the probability integral transformation for each component of the random variable vector

$$U_i = F_i(X_i) \text{ for } i = 1 \text{ and } 2. \tag{8}$$

Thus, the joint distribution function of (X_1, X_2) is is $F_{1,2}$ defined as a function of (U_1, U_2) which distributes uniformly over the interval $[0, 1]$, so

$$F_{1,2}(X_1, X_2) = C(U_1, U_2) \tag{9}$$

the C copula function (Nelsen 1999). C_U: $[0, 1]^2$ is the combined distribution of functions of the random variables transformed from $U_j = F_{X_j}(x_j)$ for $j = 1$ and 2. Because this transformation U_j always has a univariate marginal distribution (Schölzel & Friederichs, 2008).

In the bivariate case, two random variables are used to determine the correlated joint distribution function. In general, two variables are random and are said to be correlated if they are dependent. Copulas play a role in combining these dependency structures to form a combined distribution of the variables. The dependency problem in this case is that there are two variables that are distributed at extreme values. Copulas with random variables distributed at extreme values belong to the Archimedean Copula family. (Yanti, 2020)

Archimedean Copula is divided into three classes, they are Clayton Copula, Frank Copula, and Gumbel Copula. Archimedean copulas have tail dependencies that differ between Clayton Copula, Frank Copula, and Gumbel Copula which are given in the following Table 1.

Table 1. The formula of Clayton Copula, Gumbel Copula, and Frank Copula

No	Copula	$C(U_1, U_2)$	Parameter
1	Clayton	$\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-\frac{1}{\theta}}$	$\theta > 0$
2	Gumbel	$exp\left[-\left(w_1^{-\theta} + w_2^{-\theta}\right)^{\frac{1}{\theta}}\right], w_i = -\ln(u_i)$	$\theta \ge 1$
3	Frank	$-\frac{1}{\theta}\ln\left(1 + \frac{w_1 w_2}{e^{-\theta} - 1}\right), w_i = e^{-\theta u_i} - 1$	$\theta \ne 1$

Clayton Copula has the advantage of identifying the behavior of the bottom dependent tail. Gumbel copula, which is a class of Archimedean copula, has the advantage of identifying the behavior of the top dependent tail. Frank copula is a type of copula that does not have a dependent tail either at the top or bottom (Sutikno, et al. 2014).

Goodness of Fit Test
Hypothesis

The goodness-of-fit test is a test to show whether or not the assumption of data comes from a specific distribution. Mathematically, the suitability of this model is tested by hypothesis and statistical testing. The hypothesis is a statement about the distribution model of a population while statistical testing is a decision-making process whether to accept or reject a hypothesis. The form of the statement of the null hypothesis (H_0) is "data with a certain distribution" and the alternative hypothesis (H_1) is "data not distributed in a certain way". Statistical testing generally measures how close the distribution function of a model is to the empirical distribution function. If the statistical value of the statistical test on a certain distribution is smaller than a predetermined value, the decision shows that the null hypothesis is not rejected (H_0) and the data is distributed in a certain way (Klugman, et al ., 2012).

Kolmogorov-Smirnov (K-S) Test

Kolmogorov-Smirnov test is used to see the difference between the empirical distribution and the distribution model to be tested. The empirical distribution $F_n(x)$ is defined by

$$F_n(x) = \frac{n_i}{n},$$

where$i$ is a finite natural number, n_i is the number of smaller observation points equal to the th observation point $i$after all observation points are sorted from smallest to largest and n is the notation of the number of observation points. The cumulative distribution of the distribution model to be tested is denoted by F_X and defined as follows

$$D^+ = \max_{1 \le i \le n} \left\{\frac{i}{n} - F_X\left(x_{(i)}\right)\right\} \text{ and } D^- = \max_{1 \le i \le n} \left\{F_X\left(x_{(i)}\right) - \frac{i-1}{n}\right\},$$

where $x_{(i)}$ is the order statistic. K-S test statistics are defined. $D = \max(D^+, D^-)$. If the value $D$is greater than the critical value determined by the K-S table at a certain level of significance, then the distribution model tested is not suitable for representing the distribution of the observed data (Klugman, et al., 2012).

Return Period Analysis

The results of selecting the best Copula using the K-S Test method was used to determine the return period. Alam et al. (2018) says that if a variable x is more than or equal to , occurs once in Q a period, then the probability of its occurrence $Pr\left(X \ge x_Q\right) = \frac{1}{Q}$ in a certain period can be obtained in equation (10) as follows:

$$Pr\left(X \ge x_Q\right) = \frac{1}{Q} \tag{10}$$

$$Q = \frac{1}{Pr\left(X < x_Q\right)} \tag{11}$$

Yanti (2020) stated that Copula-based return period analysis was carried out by Han et al. (2018) on hydrological data to detect extreme cases. Research with return period analysis was also conducted by Zhang et al. (2012) bivariate variables for cases of extreme rainfall in Xinjiang, China. Zscheischler (2017) investigated the impact of extreme climate on crop production by analyzing the return period in the bivariate case with temperature and rainfall variables. For hydro-meteorological variables U_1 and U_2 with a marginal distribution $F_X(u_1)$ and $F_Y(u_2)$, the combined probability distribution is $F(u_1, u_2)$. For the bivariate case, the description of the return period that can be investigated is as follows

$$Q_{\{U_1 > X \text{ or } U_2 \le y\}} = \frac{1}{Pr(U_1 > X \text{ or } U_2 \le y)} = \frac{1}{1 - F_X(u_1) + C(u_1, u_2)} \tag{12}$$

RESEARCH METHOD

The method is used in this research is modelling. Data, software, and research steps are given below.

Research Data and Equipment

The data used in this paper was secondary data in the form of maximum daily rainfall and maximum temperature in Dramaga Bogor. The interval time used in the data is a period of paddy planting period (September to December) from 1984 to 2021. The data were obtained from Jawa Barat Climatology Station of Meteorology, Climatology, and Geophysical Agency (BMKG). The variables used in this study were R as the maximum daily rainfall in Dramaga Bogor in a period of paddy planting period (September to December) from 1984 to 2021 and Has the maximum daily temperature in Dramaga Bogor in one period of paddy planting period (September to December) from 1984 to 2021. The software used for data processing in this study were Minitab version 21.1 (64-bit) and Ri386 4.1.3.

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Research Steps

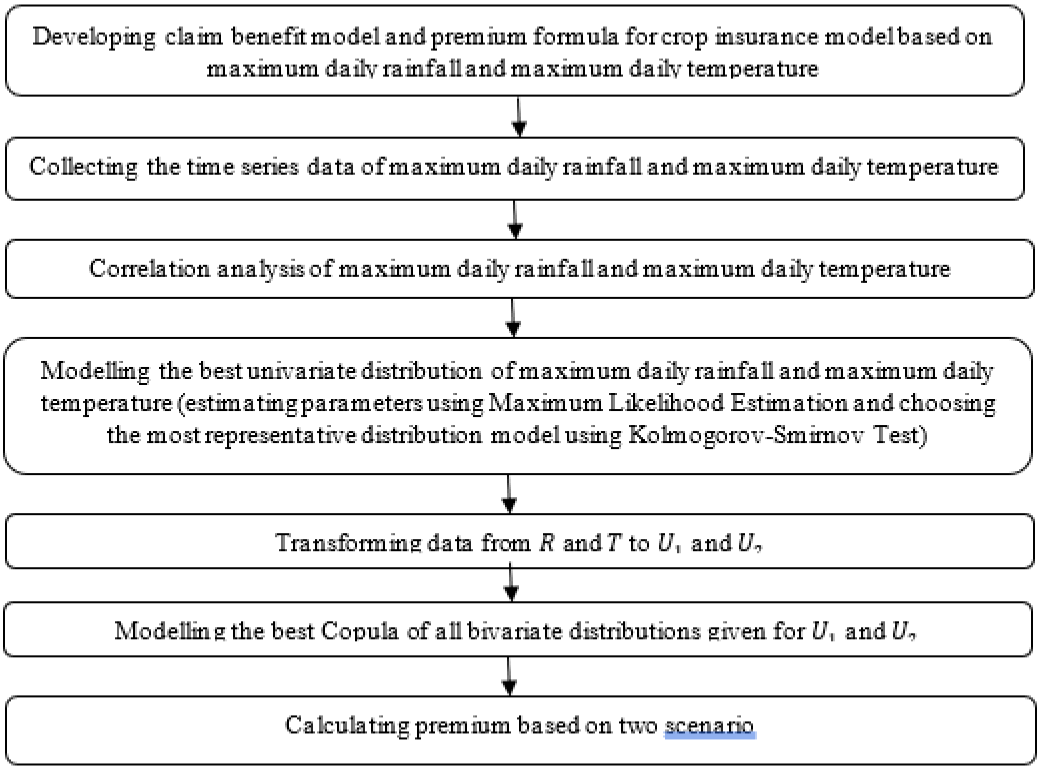

Figure 3. Research steps

RESULTS AND DISCUSSIONS

This agricultural insurance model can only be applied to rainfed land for a commodity. Agricultural commodity, such as paddy, has characteristics or growing conditions related to the need for water and temperature to reach the ideal. The crop insurance model based on maximum daily rainfall and maximum daily temperature index was developed to cover agricultural risk from total crop failure. So, it can classify risk into two conditions as follows:

I. full benefit payment for loss due to total crop failure in the amount of production cost, and

II. There is no benefit payment if the growing conditions reach the ideal.

The risk representation is explained by the parameter index. This parameter index was developed by taking into account the bivariate distribution and characteristics of the agricultural commodities to be insured.

One of the assumptions used in this model is that the maximum daily rainfall has a negative correlation with the maximum daily temperature. On the other hand, paddy has total crop failure if the rainfall and temperature are too high or low. If this is related to the two conditions of benefit payment and the previous benefit claim model, then only the exit index parameter is needed. Exit is a benchmark that contains the risk of total crop failure so that policyholders can claim full benefit. In this model, $E_{i,j}$ denotes benchmark to make a claim with i = 1 means if actual value is less than the value of this index that causes total crop failure of drought and $i = 2$ interprets if actual value is greater than the value of this index that causes total crop failure of flooding. $j = 1$ is the index for the origin of the cause of total crop failure from the maximum daily rainfall and $j = 2$ is the index for the origin of the cause of total crop failure of the maximum daily temperature. Therefore, the two payment conditions can be described in more detail as follows:

Condition I: $\boldsymbol{r \leq E_{11}}$ **or** $\boldsymbol{t > E_{12}}$ **or** $\boldsymbol{r > E_{21}}$ **or** $\boldsymbol{t < E_{22}}$

Condition 1 is a condition to payment of benefit for loss due to total crop failure in the amount of production cost. Condition 1 can be carried out on actual condition that represent drought. This condition can also be described as if the rainfall is less than a certain exit for rainfall (E_{11}) or a high maximum daily temperature exceeds a certain exit for a certain temperature (E_{12}). The probability of condition 1 can be denoted by $K_1 = Pr(r \leq E_{11} \ \cup \ t > E_{12})$. In addition, condition 1 can also be carried out on actual conditions that represent flooding. It can be described if the maximum daily rainfall is more than a certain exit for rainfall (E_{21}) or the maximum daily temperature that is less than a certain exit for the temperature with probability $K_2 = Pr(r > E_{21} \ \cup \ t < E_{22})$. The amount of the claim given is the full benefit (b).

Condition II : $\boldsymbol{E_{11} < r \leq E_{21}}$ dan $\boldsymbol{E_{12} < t \leq E_{22}}$

Condition II is an ideal condition for an agricultural commodity. In this condition, agricultural commodity does not experience total crop failure so that policyholder cannot claim benefit payments. The claim amount in condition II is 0 so that the contribution to the premium amount which is the multiplication of the claim amount with the probability is 0.

The developing benefit claim model is given in Figure 4 as follows:

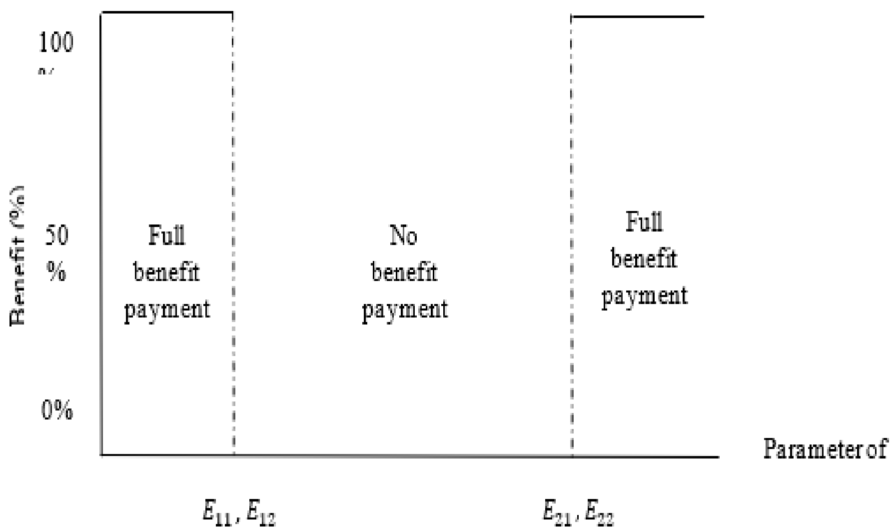

Figure 4. Illustration of the developing benefit calim model in crop insurance model based on maximum daily rainfall and maximum daily temperature

Premium

In actuarial science, premium can be calculate with the expected value of the claim amount with its probability as the weight. This premium is paid once at the beginning of the insurance contract. The premium value per hectare in IDR can be formulated by

$$P = b[K_1 + K_2]$$

$$P = b[Pr(r \leq E_{11} \ \cup \ t > E_{12}) + Pr(r > E_{21} \ \cup \ t < E_{22})]$$

Based on the return period, P it can also be written as follows:

$$P = b\left[1 - F_T(E_{12}) + C\big(F_R(E_{11}), F_T(E_{12})\big) + 1 - F_R(E_{21}) + C\big(F_R(E_{21}), F_T(E_{22})\big)\right]$$

Case Study

Modelling univariate distribution and bivariate distribution of maximum daily rainfall and maximum daily temperature were given bellow.

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Correlation Analysis

Spearman correlation test was used to calculate the correlation between the maximum daily rainfall and maximum daily temperature variables. Based on the Spearman correlation test, the correlation value is -0.069. This indicates that the maximum daily rainfall has a negative relationship with the maximum daily temperature. If the maximum daily temperature increases, the maximum daily rainfall decreases. The data qualify for applying in crop insurance model based on maximum daily rainfall and minimum daily rainfall. The following is presented in Figure 5 to see the correlation of these two variables.

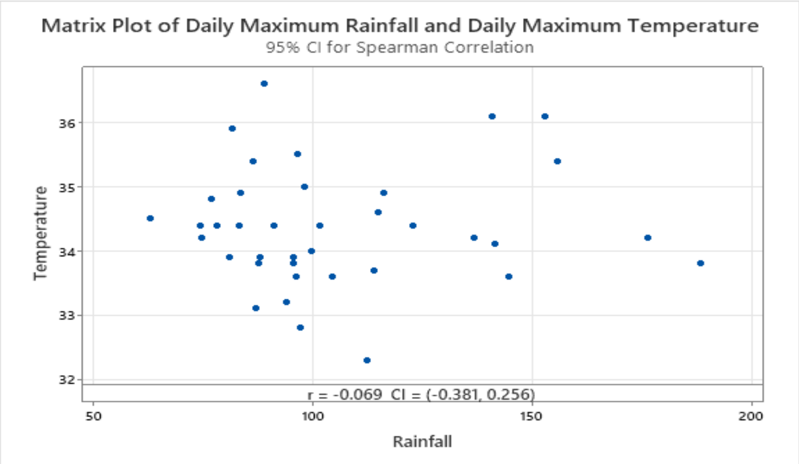

Figure 5. Correlation analysis plot

Maximum Daily Rainfall Data Exploration

Histogram of 38 observations of maximum daily rainfall for the period of one paddy planting period (September to December) was given in Figure 6. The histogram showed maximum daily rainfall data in the range of 63 mm up to 188.3 mm with an average of 105.80 mm. The maximum daily rainfall in Dramaga Bogor is categorized as very heavy rainfall. This means that Dramaga Bogor for the period from September to December in the past had the potential for flooding. Based on the asymmetric data distribution, the distribution of the maximum daily rainfall data will be modeled by the Normal distribution and GEV distribution.

Furthermore, the two models will be calibrated through the parameter estimation step.

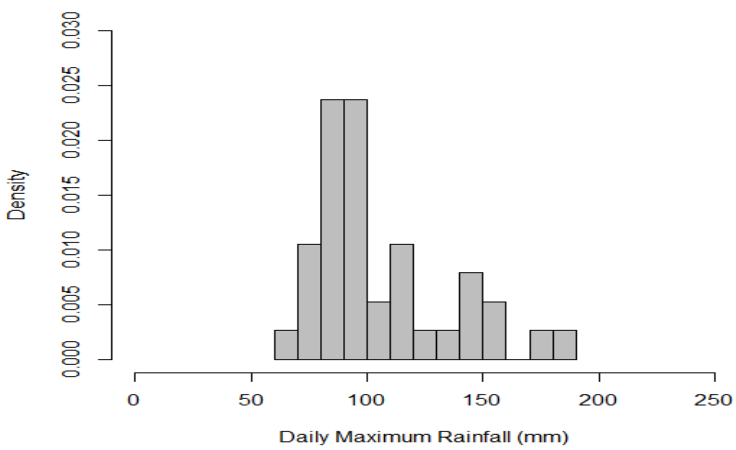

Figure 6. Histogram of maximum daily rainfall in Dramaga Bogor during paddy planting period from 1984 to 2021

Maximum Daily Temperature Data Exploration

Histogram of 38 maximum daily temperature observations for one paddy planting period (September to December) was given in Figure 7. The histogram showed maximum daily temperature data in the range of 32.3 ℃ up to 36.6 ℃ with an average of 34.36 ℃. It means that Dramaga Bogor for the period from September to December in the past had the potential for drought . Based on the data distribution, the distribution of the maximum daily rainfall data will be modeled by the Normal distribution and GEV distribution. Furthermore, the two models will be calibrated through the parameter estimation step.

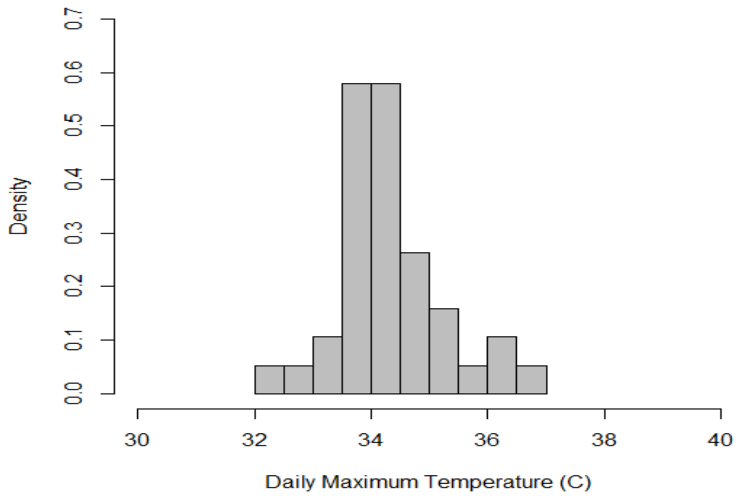

Figure 6. Histogram of maximum daily Temperature in Dramaga Bogor during paddy planting period from 1984 to 2021

Parameter Estimation and Goodness of Fit Test for Univariate Distribution

Normal and GEV distribution models were obtained by using the Maximum Likelihood Estimation method. The estimation method tries to select the parameters that make a distribution close to the data. Estimating values of parameters and statistic value of Kolomogorov-Smirnov test for each model can be seen in Table 2.

Table 2. Estimated value of parameters and statistic value of K-S test for Normal distribution and GEV distribution

No.	Variable	Type of The Result	Normal Distribution	GEV Distribution
1	Daily Maximum Rainfall	Parameter	$\mu = 105.8$ $\sigma = 29.54$	$\alpha = 91.41$ $\lambda = 19.27$ $\xi = 0.15$
		K-S Test (Statistic Value)	0.19 (< 0.22)	0.09 (< 0.22)
2	Daily Maximum Temperature	Parameter	$\mu = 34.36$ $\sigma = 0.93$	$\alpha = 34$ $\lambda = 0.86$ $\xi = -0.19$
		K-S Test (Statistics Value)	0.14 (< 0.22)	0.12 (< 0.22)

Based on Table 2, the statistic value of each variable is lower than the critical value, which is 0.22. For maximum daily rainfall, the statistic value of the Kolmogorov-Smirnov test on the GEV model is 0.09 smaller than the statistic value on the Normal model is 0.19. In addition, the statistic value of the Kolmogorov-Smirnov test of the GEV model at a maximum daily temperature of 0.12 is also

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smaller than the statistic value of the Normal model of 0.14. This indicates that the maximum daily rainfall distribution (eq. (13)) and maximum daily temperature (eq. (14)) are more representative

$$F_R(r) = \exp\left\{-\left(1 + 0.15\frac{r-91.41}{19.27}\right)^{-\frac{1}{0.15}}\right\}, \text{for } \xi \neq 0, \text{ and} \tag{13}$$

$$F_T(t) = \exp\left\{-\left(1 + (-0.19)\frac{t-34}{0.86}\right)^{-\frac{1}{(-0.19)}}\right\}, \text{ for } \xi \neq 0 \tag{14}$$

modeled by the GEV model than the Normal model.

Parameter Estimation and Goodness of Fit Test for Bivariate Distribution

Prior to testing, the data was transformed into U_1 and U_2. Then the values of the Clayton, Gumbel, and Frank Copula model parameters were obtained using the Kendall-Tau method. This method is built-in to R. The estimation method tries to select the parameters that make a distribution close to the data. Parameter estimator values for each model can be seen in Table 3 .

Table 3. Estimated value of parameters and statistic value of K-S test for Clayton, Gumbel, and Frank Copula

No.	Copula	Parameter (θ)	KS Test (Statistic Value)
1	Clayton	-0.09	0.65
2	Gumbel	1	0.69
3	Frank	-0.45	0.62

Based on table 3, the statistic value of Copula Frank is 0.62 smaller than the statistic value of Clayton Copula and Gumbel Copula. Therefore, Copula Frank is more representative to model the data than other models. It can be seen in Figure 5 that there are no dependency below and above the tail so it is appropriate to use Frank Copula. Therefore, the Copula model can be formulated with

$$C(U_1, U_2) = -\frac{1}{(-0.45)}\ln\left(1 + \frac{w_1 w_2}{e^{-0.45} - 1}\right), w_i = e^{-(-0.45)u_i} - 1 \text{ for } i = 1,2. \tag{14}$$

Illustration of Claim Benefit and Determination of Premium

Some of the assumptions used in the benefit claim model as follows:

a. The parameter index was formed by maximum daily rainfall and maximum daily temperature during one paddy planting period, from September to December in Dramaga Bogor during 38 years. Distribution of each variable followed historical distribution function. Based on the results, maximum daily rainfall is better to represent by GEV distribution, as well as maximum daily temperature is better to represent by GEV distribution. Furthermore, joint cumulative distribution function is the best to represent by Frank Copula of the others.

b. Policyholders paid premium once at the beginning of the contract insurance. Then policyholders claimed benefit at the end of the period. Full benefit was IDR 6 million per hectare which was proportional with paddy production cost, and

c. Illustration used characteristics of paddy with probability of risk based on scenario cases (1 and 2) in the Table 4 as follows.

Table 4. Illustration of premium calculation with certain exit value scenario

No	Scenario	Premium (IDR/Hectare)
1	$E_{11} = 85.27$ mm $E_{12} = 35$°C $E_{21} = 163.52$ mm $E_{22} = 20$°C	2 482 925

2	E_{11} = 68.74 mm E_{12} = 35°C E_{21} = 163.52 mm E_{22} = 20°C	1 495 660

Based on Table 4, for first scenario, a policyholder has the right to claim the full benefit of IDR 6 million if the maximum daily rainfall during one paddy planting period is less than 85.27 mm or more than 163.52 mm or if the maximum daily temperature during one paddy planting period is less than 20 ℃ or 35 ℃. The selection of rainfall exit takes into account the quantile by providing the probability to claim in case of drought by 25% and flooding by 5%. For the exit temperature, the ideal condition for paddy growing is at 20 ℃ to 35 ℃. Therefore, in case 1, the premium price is IDR 2 482 925/hectare.

For second scenario, the treatment is carried out by slightly changing the rainfall exit in the case of drought. Change is made by taking into account the quantile of 2.6% which is the return period for one possible benefit claim for 38 years happened from the actual maximum daily rainfall which is less than the exit. In the second scenario, the premium price is IDR1 495 660/hectare. Therefore, the smaller the chance of claiming benefit , the lower the premium price. On the contrary, the greater the chance of claiming benefit. the higher the premium.

CONCLUSION

As the result, crop insurance model can be developed by two variables of index, such as maximum daily rainfall and maximum daily temperature. To show this model, the marginal function is needed for maximum daily rainfall and maximum daily temperature, respectively. Then, each joint density function for these two variables is determined. In this research, for benefit claim model, characteristics of particular agricultural commodity is required to construct the parameters of index. Those parameters are developed to classify benefit model into two condition, full benefit condition and no benefit condition. The case study showed that the marginal function of maximum daily rainfall available was considered by GEV distribution. The same result came from maximum daily temperature that followed GEV distribution. The joint density function for this bivariate model was better to be represented by Frank Copula model. Furthermore, the value of premium could be calculated by the formula given and was lower than the total benefit that policyholder will be claimed.

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