

Modal Analysis of Diagonal Arch Bridge with Simple Supports (Pinned–Roller) and Lead Rubber Bearings (LRB)

Nazar Nazar, Akhmad Aminullah, Bambang Suhendro

Department of Civil and Environmental Engineering, Universitas Gadjah Mada, Yogyakarta, INDONESIA

E-mail: nazar@mail.ugm.ac.id

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ABSTRACT

Diagonal arch bridges exhibit unique structural behavior due to their asymmetrical hanger arrangements and complex dynamic interactions among the arch, girders, and hangers. While conventional pinned–roller supports are widely used for their simplicity and cost efficiency, their high horizontal stiffness leads to shorter natural periods and greater seismic demands. To enhance seismic performance, this study investigates the use of lead rubber bearings (LRB) as an alternative support system and evaluates their effects on the dynamic characteristics of a diagonal arch bridge. Using the Detail Engineering Design (DED) of the Mungkid Bridge in Indonesia, two finite element models were developed in Midas Civil 2025: one with conventional pinned–roller supports and another with LRB isolators. Modal analysis was conducted to determine natural periods, mode shapes, and mass participation. Results show that incorporating LRBs lengthens the bridge's fundamental period by up to 182% compared with the simple-support configuration, effectively reducing natural frequencies and seismic acceleration demands. The LRB model also concentrated modal mass in the first few modes, enabling clearer identification of dominant vibration mechanisms. Although the isolators introduce greater flexibility and improve seismic resilience, further nonlinear time-history analysis is recommended to capture hysteretic effects. These findings highlight the potential of LRBs to improve the seismic adaptability of diagonal arch bridges.

Keywords: diagonal arch bridge, modal analysis, LRB, simple support.

INTRODUCTION

The diagonal arch bridge represents a distinctive structural typology characterized by its unique and complex features. Its characteristic arch form is combined with a diagonal system connecting the main girders, resulting in each support being positioned on opposite sides of the deck[1]. This concept emerged from the growing tendency of architecture to intervene in the domain of structural engineering, prioritizing formal expression and aesthetic value over conventional technical reasoning. However, such architectural exploration also introduces new challenges in structural behavior, particularly as its asymmetric configuration produces more complex dynamic responses, especially under seismic loading[2]. In practice, simple supports such as pinned–roller are still widely adopted because of their ease of implementation and cost efficiency. However, such supports maintain relatively high horizontal stiffness, which leads to shorter natural periods and higher inertial forces during earthquakes [3]. As an alternative, seismic isolation systems such as the Lead Rubber Bearing (LRB) have been developed to provide additional flexibility while also dissipating energy. The LRB combines layers of rubber with a lead core, enabling it to lengthen the natural period of the structure, reduce peak acceleration, and increase damping capacity[4],[5]. Its effectiveness has been demonstrated across various types of bridges, including box girder bridges[6], the Kutai Kartanegara steel arch bridge[7], and the Kalikuto Bridge[8].

Modal analysis is a fundamental method for understanding the dynamic characteristics of structures. Through the eigenvalue approach, it enables the evaluation of natural periods, frequencies, and mode shapes, allowing the identification of dominant vibration modes that are crucial for assessing structural stability and resonance potential [9], [10], [11]. These characteristics are highly influenced by the support system, as changes in stiffness can shift natural frequencies and alter the overall dynamic response of the structure. Nevertheless, studies that specifically investigate the influence

of different support systems on the modal characteristics of diagonal arch bridges remain very limited. To address this gap, the present research introduces novelty by analyzing and comparing diagonal arch bridges with two different support systems, namely pinned–roller and the Lead Rubber Bearing (LRB). The findings are expected to provide deeper insights into the effects of support variation on dynamic responses, thereby contributing to the development of bridge design strategies that are more adaptive and resilient to seismic loading.

RESEARCH METHODS

Bridge Technical Data

The data used in this study are based on the Detail Engineering Design (DED) of the Mungkid Bridge, which is part of the Yogyakarta–Bawen toll road and located at the Mungkid interchange. The bridge has two main spans of 51.42 m and 46.46 m, with a total length of 97.88 m. It consists of two diagonal arches, each with heights of 35 m and 50 m, as illustrated in Figure 1.

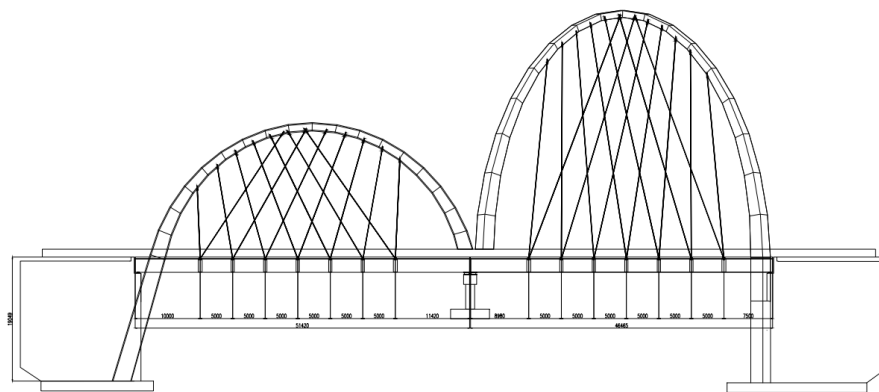


Figure 1. Structural Layout of the Mungkid Bridge

The bridge employs concrete with a specified compressive strength of $f'_c = 30$ MPa. The structural steel follows ASTM A572 Grade 50, with a yield strength of $f_y = 350$ MPa, while the hangers utilize ASTM A416 strands with an ultimate tensile strength of $f_{pu} = 1860$ MPa. Other parameters, including Poisson's ratio, modulus of elasticity, and unit weight, are adopted in accordance with the prevailing standards in Indonesia.

Methods

This study employs a finite element–based numerical analysis to evaluate the influence of conventional simple supports (pinned–roller) and seismic isolators in the form of Lead Rubber Bearings (LRB) on the dynamic characteristics of a diagonal arch bridge. The structural model was developed in Midas Civil 2025 using technical data and the Detail Engineering Design (DED) as the primary reference. The arch ribs and main girders were represented by beam elements to capture both flexural and axial behavior, while the hangers were modeled as tension-only elements to accurately represent the unidirectional tensile capacity of cables. The analysis was restricted to the superstructure, deliberately excluding the substructure to simplify the modeling domain.

Two bridge models were established for comparative analysis. The first model adopted a conventional support system with pinned–roller bearings, while the second employed LRB supports, whose parameters were defined according to AASHTO [17] provisions. Dynamic characteristics were assessed through an eigenvalue analysis to extract the natural periods, mode shapes, and mass participation ratios. The evaluation of natural periods was limited to the first eleven modes to capture the dominant structural responses, with particular attention given to four primary mode-shape categories: transverse translation, longitudinal translation, vertical translation, and rotation. To ensure adequate representation of the global dynamic behavior, the cumulative mass participation was extended to the first 100 modes, following the AASHTO [17] requirement that at least 90% of the total structural mass be activated in the modal analysis.

The numerical results from both models were compared quantitatively in terms of natural periods and mass participation distributions, and qualitatively through the observation of deformation patterns in the mode shapes. Validation was conducted using a phenomenological approach by comparing the numerical trends with findings from previous experimental studies. Referring to studies on steel and concrete bridges in Indonesia [12], [13] the natural frequencies and mode shapes identified through finite element modeling were found to be consistent with ambient vibration test (AVT) results, with frequency deviations generally within $\pm 10\%$. Therefore, the outcomes of this study are considered validated when exhibiting similar trends, particularly the consistency of global mode shapes, the dominance of low-order modes, and the tendency of natural period variations that align with the expected behavior of elastomeric bearing systems such as LRB, without implying exact correspondence with the bearing types used in the referenced literature

Bridge Modeling

The numerical modeling of the diagonal arch bridge in this study was limited to the superstructure, which consists of the girders, arch ribs, and hangers. The substructure components, including the abutments, piers, and pier heads, were not modeled in either Model A or Model B. The types of elements employed in both bridge models are presented in Table 1.

Table 1. Bridge Modeling Elements

Bridge Component	Element Modeling
Arch rib	Beam element - tapered section
Composite I-Girder	Beam element
Cross Girder	Beam element
Hanger	Tension only
Lead Rubber Bearing	Elastic link (K_1)
Arch Support	Rigid support

In Model A, the bridge supports were represented as pinned–roller bearings, placed at both ends of the bridge span. The pinned support restrains translation in all three directions (X, Y, Z) while allowing rotation, whereas the roller support restrains translation in the Y and Z directions but permits rotation in all axes, as illustrated in Figure 2.

In Model B, the supports were modeled using Lead Rubber Bearings (LRB), represented as two-node link elements arranged in parallel, with stiffness parameters idealized as elastic link materials. The LRBs were assigned at both ends of each girder span, with a total of 40 units, where each span was equipped with 20 units. Each LRB has a diameter of 400 mm, with linear stiffness of 5.24 kN/mm in both longitudinal and transverse directions, whereas the vertical stiffness is significantly higher than the horizontal stiffness. The LRB properties used in this model were taken from the Fresineer catalog, as summarized in Table 2.2, and the corresponding modeling scheme is presented in Figure 2.

Table 2. Properties of Lead Rubber Bearings

$\varnothing$ (mm)	T_r (mm)	H (mm)	Δ_{max} (mm)	V_{max} (kN)	V_{seism} (kN)	K_2 (kN/mm)	K_1 (kN/mm)	F_y (kN)	K_{eff} (kN/mm)	ξ (%)
400	90	192	225	1950	630	0.52	5.24	60	0.77	26

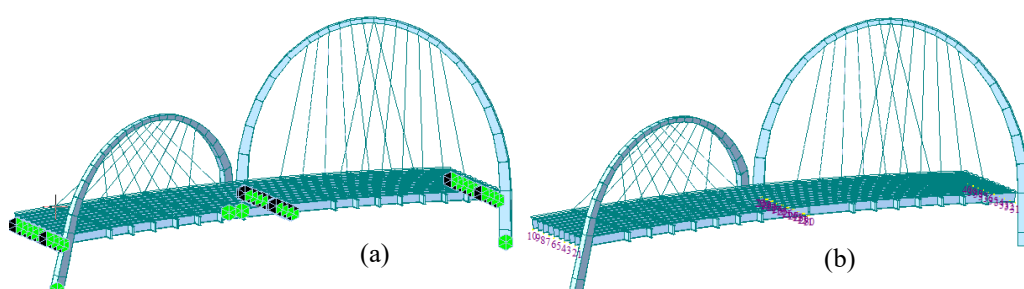


Figure 2. Finite Element Model of the Bridge: (a) Simple Supports, (b) Lead Rubber Bearings

A modal analysis was then carried out to evaluate the natural periods, mode shapes, and mass participation of both bridge models. This analysis was performed using the eigenvalue method without considering external loads, under the assumption that the structure behaves as a linear elastic system, in accordance with common practices in bridge dynamic analysis.

RESULT AND DISCUSSION

The results of the modal analysis using the eigenvalue method for the diagonal arch bridge with two support systems, namely simple supports and LRB, are presented below. The outcomes include the natural periods, mode shapes, and mass participation, which are illustrated both numerically and visually.

Natural Period

The analysis indicates that the use of LRBs significantly lengthens the natural periods of the bridge. For the first mode, the natural period of the simply supported model was 0.948 s, whereas the LRB model reached 1.725 s, representing an increase of approximately 182 % as shown in Table 3. Similar trends were observed for the second and third modes, with increases of 192 % and 193 %, respectively, compared with the simple-support configuration. This extension of the natural period reflects the reduced lateral stiffness provided by the isolators, shifting the natural frequencies to lower values. Such a shift is consistent with the fundamental principle of seismic isolation, where a longer vibration period reduces seismic acceleration demands and consequently lowers inertial forces acting on the structure [14] [15].

Table 3. Eigenvalue analysis results of the diagonal arch bridge with simple supports and LRB

Mode	Simple Support		Lead Rubber Bearing		Difference	
	Frequency (cycle/sec)	Period (sec)	Frequency (cycle/sec)	Period (sec)	Frequency	Period
1	1.054749	0.948093	0.579498	1.725631	55%	182%
2	1.16098	0.861341	0.606157	1.649737	52%	192%
3	1.327622	0.753227	0.689595	1.450128	52%	193%
4	1.749677	0.571534	0.693302	1.442374	40%	252%
5	1.877425	0.532645	1.004857	0.995166	54%	187%
6	2.019992	0.495051	1.035124	0.966068	51%	195%
7	2.912046	0.343401	1.226772	0.815147	42%	237%
8	3.128996	0.319591	1.746726	0.5725	56%	179%
9	3.273658	0.305469	1.854034	0.539364	57%	177%
10	3.466474	0.288478	2.036895	0.490943	59%	170%
11	3.597641	0.27796	3.174566	0.315004	88%	113%

Mode Shapes

The visualization of the first mode shows that the deformation pattern dominated by Y-direction translation, representing a uniform lateral displacement of the entire bridge deck. The response resembles a global lateral sway with no evidence of torsional rotation. The simply supported model shows a stiffer lateral movement because the supports provide no additional flexibility Figure 3,

whereas the LRB model allows a more compliant lateral displacement due to the isolators' horizontal flexibility Figure 4. This behavior is consistent with the findings of [16], who reported that elastomeric bearings enhance horizontal flexibility and influence lateral deformation patterns in fundamental modes.

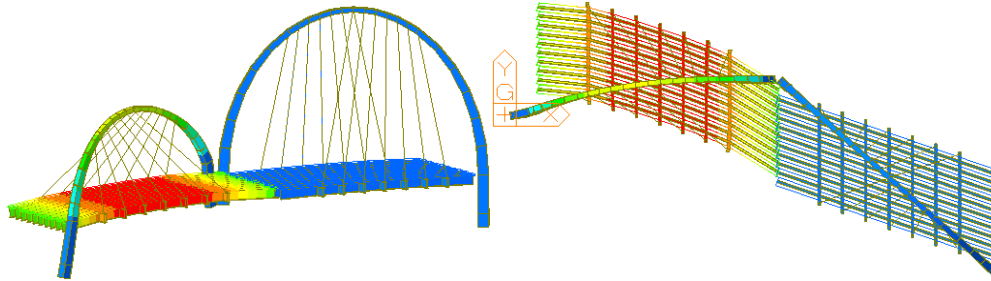


Figure 3. Mode 1 – Simple support model, period 0.948 seconds

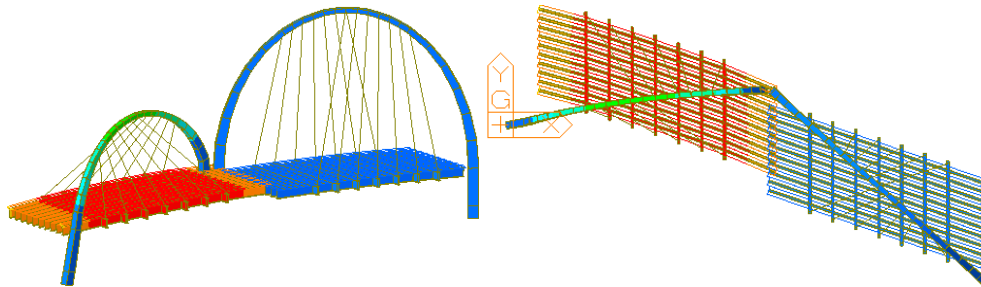


Figure 4. Mode 1 – LRB model, period 1.725 seconds

The second mode remains primarily controlled by Y-translation but introduces a noticeable Z-rotation. In the simply supported model, lateral translation and torsion contribute almost equally Figure 5, while in the LRB model Y-translation continues to dominate even though Z-rotation becomes more pronounced Figure 6.

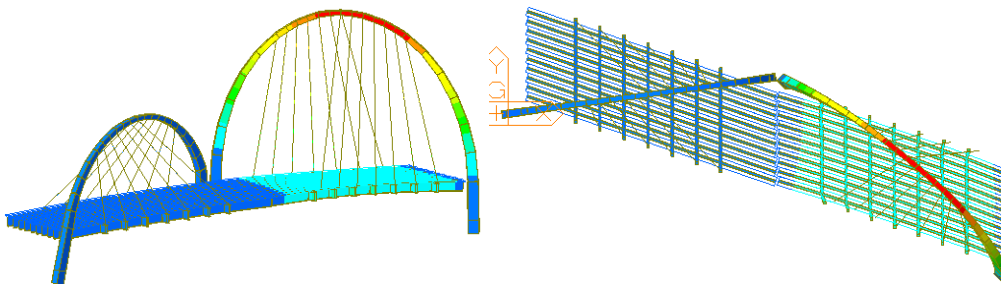


Figure 5. Mode 2 – Simple support model, period 0.861 seconds

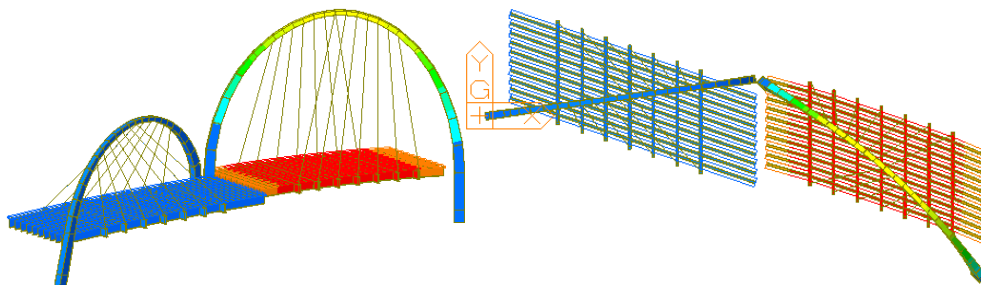


Figure 6. Mode 2 – LRB model, period 1.692 seconds

The third mode presents a more complex deformation pattern. For the simply supported system, Y-translation remains significant but is accompanied by stronger Z-rotation, resulting in lateral motion

combined with appreciable torsion Figure 7. For the LRB model, an additional X-translation (longitudinal motion) emerges alongside Y-translation and Z-rotation Figure 8, indicating that the isolation system engages a coupled lateral–longitudinal–torsional response in this higher mode.

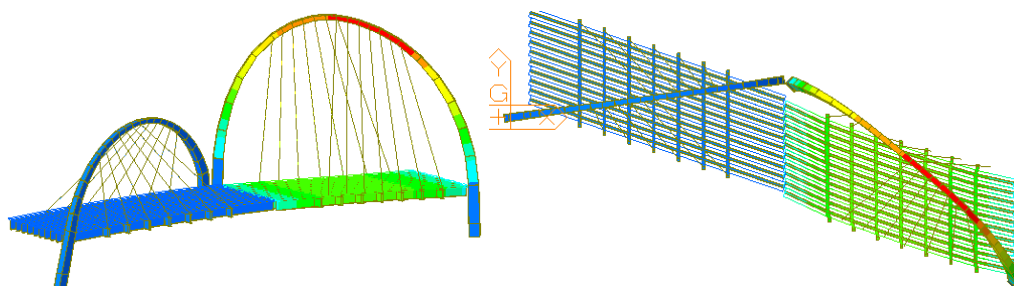


Figure 7. Mode 3 – Simple support model, period 0.753 seconds

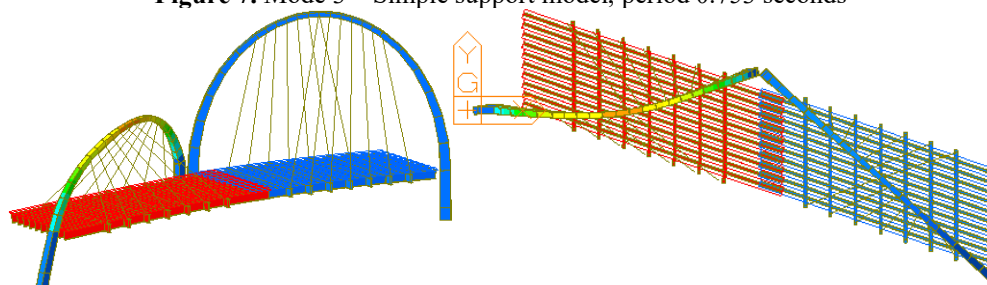


Figure 8. Mode 3 – LRB model, period 1.492 seconds

The rotational mode in the simply supported (pinned–roller) system exhibits dominant motion along the X-longitudinal and Z-vertical axes, indicating a significant involvement of torsional components Figure 9. This pattern reflects a coupling between rotational and lateral deformations induced by the movement constraints at the supports, resulting in a stiffer rotational response with limited damping capacity. This condition emphasizes that the rotational mode is not purely torsional but influenced by the interaction between the structure's torsional and lateral stiffness. In contrast, within the Lead Rubber Bearing (LRB) system, the rotation about the Z-vertical axis develops more independently with higher damping capacity due to the deformability of the elastomeric layers and the hysteretic energy dissipation of the lead core Figure 10. Consequently, the structure demonstrates a more flexible, stable, and dynamically resilient rotational behavior, confirming the effectiveness of the isolation system in reducing both torsional and rotational responses under seismic excitation.

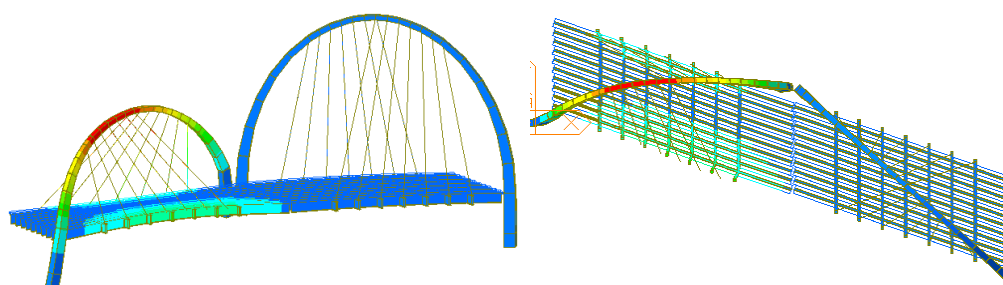


Figure 9. Mode 6 – Simple support model, period 0.495 seconds

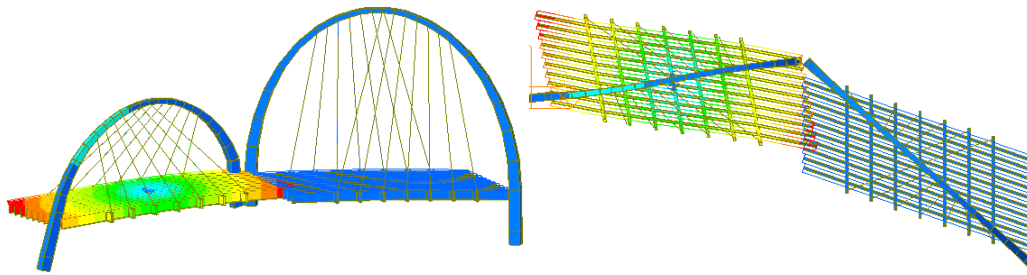


Figure 10. Mode 5 – LRB model, period 0.995 seconds

The vertical mode in the simply supported system exhibits dominant motion along the Z-vertical axis, accompanied by noticeable rotational components about the X-longitudinal and Y-transverse axes. This pattern indicates a coupling between vertical deformation and rotational motion influenced by the stiffness distribution and boundary constraints at the supports. In the Lead Rubber Bearing (LRB) system, the vertical mode remains governed by motion along the Z-axis, with minor rotational participation around the X and Y axes still observed. These findings suggest that the vertical response in both systems is not entirely independent, but rather reflects an inherent interaction between bending and rotational mechanisms that shape the global dynamic behavior of the structure.

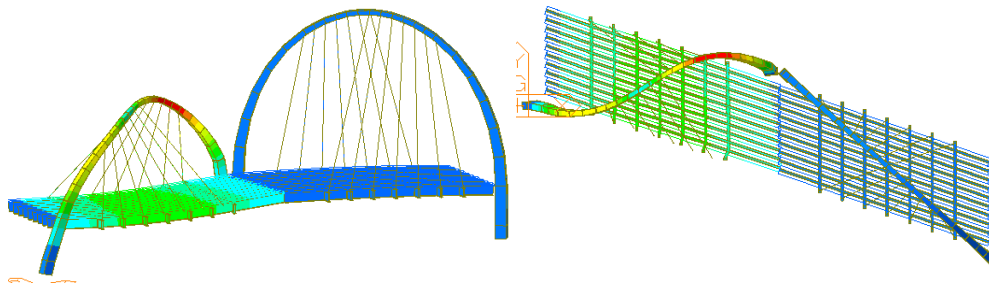


Figure 11. Mode 8 – Simple support model, period 0.319 seconds

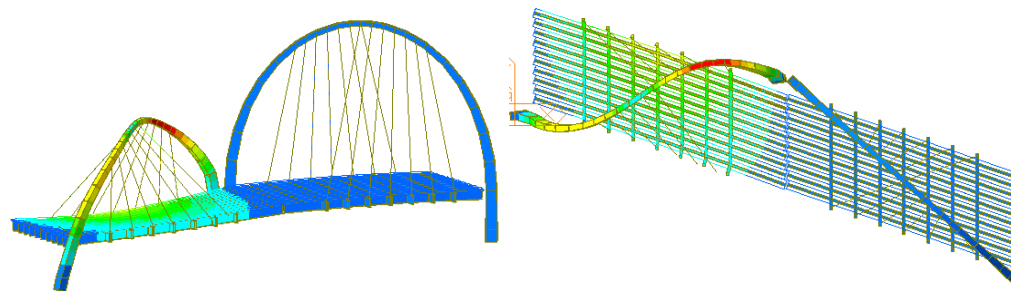


Figure 12. Mode 11 – LRB model, period 0.315 seconds

Mass Participation

The distribution of modal mass highlights clear differences between the two support systems. For the simple-support model, X-direction mass participation reached 39.25 % in the first mode, increased to 58.74 % in the second, and 74.43 % in the third. Cumulative mass participation of 93.33 % was achieved only after approximately 100 modes show Table 4, indicating that the structural response is multi-mode dominated.

For the LRB model, the longitudinal modal mass participation reached 37.80% in the first mode, rose sharply to 72.99% in the second mode, and 81.66% in the third mode. A cumulative mass ratio of 90% was already achieved by mode eight, and a total of 95.46% was reached by the 100th mode, demonstrating a low-mode dominated behavior in which most of the dynamic energy is concentrated in only a few lower modes.

This concentration of mass in the lower modes is consistent with previous findings [16], where the flexibility of rubber bearings was shown to reduce natural frequencies and lengthen vibration periods, thereby increasing the mass contribution of the fundamental modes and reducing the number of modes required to achieve an adequate cumulative mass ratio. Although the dominance of the lower modes facilitates the identification of critical modes, it does not necessarily simplify the overall dynamic assessment, because the nonlinear behavior of the isolators and the dynamic interactions between modes can only be evaluated through a more detailed time-history analysis.

Table 4. Mass participation of the simple support model

No Mode	Translation X Mass(%)	Translation Y Mass(%)	Translation Z Mass(%)	Rotation X Mass(%)	Rotation Y Mass(%)	Rotation Z Mass(%)
1	4.14	39.25	0.00	2.23	0.14	28.63
2	7.87	58.74	0.02	9.49	0.90	47.94
3	10.13	74.43	0.02	24.04	1.24	59.65
4	10.56	75.11	0.03	24.75	1.53	60.24
5	11.74	77.48	0.15	25.30	5.58	61.47
6	11.97	78.65	0.40	30.79	5.69	64.02
7	11.99	79.02	2.13	31.47	7.47	67.62
8	14.41	79.07	12.33	37.13	14.23	68.61
9	14.84	79.53	12.40	37.19	14.45	68.72
10	15.59	80.16	16.07	38.04	16.18	76.42
11	18.20	80.33	34.07	44.53	30.42	77.37
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70	39.23	87.99	50.68	54.76	42.54	89.89
71	40.56	88.11	50.82	54.90	43.18	90.04
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99	74.79	93.15	57.11	57.40	46.07	92.23
100	76.54	93.33	57.45	57.61	46.90	92.26

Table 5. Mass participation of the lead rubber bearing (LRB) model

No Mode	Translation X Mass(%)	Translation Y Mass(%)	Translation Z Mass(%)	Rotation X Mass(%)	Rotation Y Mass(%)	Rotation Z Mass(%)
1	7.39	37.80	0.00	2.90	0.20	28.83
2	15.44	72.99	0.00	4.27	0.21	59.60
3	52.32	81.66	0.00	4.54	1.07	60.06
4	86.34	87.80	0.00	7.03	1.42	60.18
5	86.35	87.80	0.07	7.06	1.49	73.07
6	86.35	87.82	0.07	7.08	1.49	83.24
7	86.79	89.63	0.08	25.22	3.15	84.30
8	87.44	90.07	0.09	26.23	3.61	84.81
9	88.89	91.10	0.20	27.01	7.31	85.91
10	89.29	92.13	0.41	33.40	7.33	87.21
11	89.34	92.13	10.60	39.23	13.75	87.94
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99	94.98	95.42	59.99	59.48	50.45	92.65
100	94.98	95.46	59.99	59.53	50.52	92.66

Structural Implications and Numerical Validation

The analysis results indicate that the implementation of Lead Rubber Bearings (LRBs) in the diagonal arch bridge significantly enhances the structure's dynamic performance. This improvement is reflected in the elongation of the natural period, which effectively reduces seismic acceleration

and inertial forces transmitted to the superstructure. Validation through Nonlinear Time History Analysis (NLTHA) revealed that most of the seismic energy was effectively absorbed by the LRBs, with a maximum shear displacement of 145.13 mm, well below the LRB design capacity of 225 mm. This finding confirms the effectiveness of the isolation system in mitigating seismic response and enhancing the bridge's overall dynamic resilience. Furthermore, the accuracy of the Finite Element (FE) modeling approach employed in this study is supported by phenomenological validation based on the results from prior research [4] [12] [13]. These studies specifically performed ambient vibration tests (AVT) to verify the FE models of several steel bridges in Indonesia. Their results indicated that the natural frequencies obtained from FE modeling deviated by only about $\pm 10\%$ from the field AVT measurements. Although the support conditions in those bridges were not specified, the close agreement between numerical predictions and field measurements substantially corroborates the reliability of the FE modeling approach employed in the present study. Consequently, the findings of this study such as the predicted period elongation and stable mode shapes can be considered numerically validated.

CONCLUSION

The use of lead rubber bearings (LRB) significantly increased the natural period of the diagonal arch bridge to more than twice that of the simple-support system. In both configurations, the first mode remained a transverse translation; however, the LRB model exhibited a more flexible deformation pattern due to bearing displacement, whereas the simple-support model showed a stiffer lateral response. Mass participation in the simple-support bridge was distributed across many modes (multi-mode dominated), while in the LRB system it was concentrated in the first two to three modes (low-mode dominated). Although this concentration facilitates identification of dominant modes, it does not simplify the overall dynamic assessment, as verification of isolator performance still requires follow-up nonlinear time-history analysis to capture hysteretic and coupling effects.

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