

## **Concrete Pictorial Abstract (CPA) Method-Based Intervention to Improve Understanding of Two-Digit Addition Concepts in Children with Mathematics Learning Difficulties**

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### **Abstract**

Mathematics learning difficulties in elementary school children remain a critical issue due to their impact on cognitive development, academic readiness, and emotional regulation. This study emphasizes the importance of instructional approaches that align with children's cognitive developmental stages, particularly for those who have not yet fully reached concrete operational thinking despite having adequate intellectual potential. This study aimed to describe the cognitive and affective profile of a child with mathematics learning difficulties and to analyze changes in conceptual understanding following the implementation of the Concrete-Pictorial-Abstract (CPA) approach in two-digit arithmetic. A qualitative case study design was employed, involving psychological assessment, observation of learning behavior, cognitive testing, error analysis, and a four-stage CPA intervention. Data were analyzed thematically to identify patterns of difficulty, thinking processes, and learning changes before and after the intervention. The findings showed an increase in calculation accuracy from 41.6% to 83%, accompanied by reduced mathematics anxiety, increased learning engagement, and the emergence of basic mental calculation strategies. Follow-up results indicated an 80% retention of skills, suggesting that the conceptual understanding developed through the CPA stages was relatively stable. Overall, this study highlights the role of a developmentally appropriate, concrete visual abstract learning sequence in supporting both cognitive and affective aspects of mathematics learning in children with specific learning difficulties. Further research is recommended to explore the application of this approach in broader educational contexts.

**Keywords:** Concrete Pictorial Abstract; Mathematics Learning Difficulties; Mathematics Anxiety.

### **Abstrak**

*Kesulitan belajar matematika pada anak sekolah dasar tetap menjadi isu kritis karena dampaknya terhadap perkembangan kognitif, kesiapan akademis, dan regulasi emosi. Studi ini menekankan pentingnya pendekatan pembelajaran yang selaras dengan tahapan*

*perkembangan kognitif anak, khususnya bagi mereka yang belum sepenuhnya mencapai pemikiran operasional konkret meskipun memiliki potensi intelektual yang memadai. Studi ini bertujuan untuk mendeskripsikan profil kognitif dan afektif seorang anak dengan kesulitan belajar matematika dan menganalisis perubahan pemahaman konseptual setelah penerapan pendekatan Konkret-Bergambar-Abstrak (CPA) dalam aritmatika dua digit. Data kuantitatif dan kualitatif digunakan, yang melibatkan penilaian psikologis, observasi perilaku belajar, pengujian kognitif, analisis kesalahan, dan intervensi CPA empat tahap. Data dianalisis secara tematik untuk mengidentifikasi pola kesulitan, proses berpikir, dan perubahan pembelajaran sebelum dan sesudah intervensi. Temuan menunjukkan peningkatan akurasi perhitungan dari 41,6% menjadi 83%, disertai dengan penurunan kecemasan matematika, peningkatan keterlibatan belajar, dan munculnya strategi perhitungan mental dasar. Hasil tindak lanjut menunjukkan tingkat retensi keterampilan sebesar 80%, yang menunjukkan bahwa pemahaman konseptual yang dikembangkan melalui tahapan CPA relatif stabil. Secara keseluruhan, penelitian ini menyoroti peran rangkaian pembelajaran visual abstrak konkret yang sesuai dengan perkembangan dalam mendukung aspek kognitif dan afektif pembelajaran matematika pada anak-anak dengan kesulitan belajar spesifik. Penelitian lebih lanjut direkomendasikan untuk mengeksplorasi penerapan pendekatan ini dalam konteks pendidikan yang lebih luas.*

**Kata Kunci:** Concrete Pictorial Abstract (CPA); Kesulitan Belajar Matematika; Kecemasan Matematika.

## INTRODUCTION

Difficulties in learning mathematics are a significant problem at the elementary school level, often reported in educational psychology literature. About 20–30% of elementary students experience persistent difficulties with basic operations such as addition and subtraction, even with adequate intelligence (Lein et al., 2020; Kroesbergen et al., 2023). These difficulties often come with high mathematics anxiety, low engagement, and avoidance, which can harm academic readiness and emotional regulation (Zhou et al., 2020). The findings highlight the need for early identification and appropriate interventions in children's mathematical learning.

Mathematics learning requires processing abstract symbols, recognizing numerical relationships, and applying logical reasoning. Children who are not ready for abstract thinking often struggle with symbolic mathematical concepts (Chen, 2024). Many studies show that mathematical difficulties often occur when children's cognitive development does not match the demands of formal instruction (Barbieri et al., 2020; Holmes et al., 2021). According to Piaget, elementary students are in the concrete operational stage. Logical

thinking is beginning, but still depends on concrete objects and real-life experiences (Marinda, 2020). If this stage is not achieved, children may struggle to link concrete quantities to symbols, perform two-digit addition, or follow multistep numerical procedures.

Despite increased research on mathematical learning difficulties, there is a notable gap in case studies examining the specific link between concrete operational stage maturity and challenges in processing mathematical symbols among children with average to high intellectual potential. Existing research often focuses on low intelligence or neurological disorders as causes, overlooking attention, concentration, and meaningful association abilities as contributing factors (Grigorenko et al., 2020). This study addresses this gap by investigating persistent arithmetic difficulties and error patterns in children with adequate intellectual capacity (Santrock, 2011).

This research is urgent because mismatches between children's cognitive readiness and academic demands affect their math performance, especially in two-digit addition. Educators and parents need a clearer view to design learning strategies that match developmental needs. Studying cognitive and affective aspects, such as anxiety and motivation, provides a fuller picture of children's math learning.

Therefore, this study specifically aims to examine how the maturity of the concrete operational stage relates to mathematics learning difficulties in a child with average to high intellectual potential. It analyzes the impact of the Concrete–Pictorial–Abstract (CPA) approach on conceptual understanding of two-digit addition, focusing on cognitive and affective profiles, error patterns, learning behaviors, and symbol-processing limitations. This targeted exploration contributes to refining interventions for mathematical learning difficulties in children with sufficient intellectual potential.

## **RESEARCH METHOD**

This study employed a qualitative case study design (Hasibuan et al., 2022; Hasibuan & Rahmawati, 2022). Qualitative and quantitative data are used selected to obtain an in-depth understanding of the cognitive profile, learning behavior, and conceptual development of mathematics in a child with learning difficulties, as well as to analyze the learning process and changes that occurred during the implementation of the Concrete–Pictorial–Abstract (CPA) intervention. Rather than aiming to statistically test effectiveness, this study focused on exploring the dynamics of learning, error patterns, and cognitive–affective changes within a real educational and psychological context.

The study was conducted in Medan, Indonesia, with the assessment and intervention process taking place from September 10 to September 27, 2025. The intervention was carried out in an educational and clinical psychology practice setting, involving direct observation of mathematical learning behavior, interviews, and structured individual learning sessions. The research participant was an 11-year-old boy (initial H), a fourth-grade elementary school student who demonstrated specific difficulties in basic mathematical concepts, particularly in symbol recognition, two-digit addition, and numerical information processing. The participant was selected using purposive sampling based on criteria indicating average to above-average intellectual potential, accompanied by persistent difficulties in numerical learning, consistent with the characteristics of specific learning difficulties in mathematics.

Data collection involved multiple qualitative techniques, including semi-structured interviews with parents and teachers, systematic observations of learning behavior, and psychological assessments. Cognitive assessment tools included the Wechsler Intelligence Scale to evaluate general intellectual functioning and the Colored Progressive Matrices (CPM) test to assess visual-spatial reasoning ability. In addition, analysis of the child's calculation errors and problem-solving responses was conducted to identify patterns of conceptual misunderstanding. Field notes were recorded throughout the CPA intervention sessions, which were organized into concrete, pictorial, and abstract stages, to document behavioral responses, learning strategies, and emotional reactions.

To enhance data credibility, triangulation was applied by comparing findings from assessments, observations, field notes, and reports from parents and teachers. Data analysis used qualitative thematic analysis to identify recurring patterns in mathematical difficulties, cognitive processing strategies, and changes in learning behavior before, during, and after the CPA intervention. Quantitative indicators, such as pre-test and post-test scores, were used descriptively to support qualitative interpretations of learning progress rather than as a basis for statistical inference. The analytical framework was grounded in Piaget's theory of cognitive development and the concrete visual abstract learning sequence. All analyses were conducted manually due to the narrative and observational nature of the data.

## RESULTS AND DISCUSSION

### Implementation of CPA Counseling and Intervention Program

**Table 1.**  
**Summary of Pre Post Intervention Outcomes Using the CPA Method**

Aspect Evaluated	Indicator	Pre-Intervention	Post-Intervention
Mathematical performance	Accuracy (12 items)	41.6% (5/12 correct)	83% (10/12 correct)
	Dominant error type	Subtraction with borrowing, mixed operations	Minor subtraction errors
	Calculation strategy	Finger counting, tally marks	Mental calculation
Conceptual understanding	Symbol–meaning connection	Weak, inconsistent	Stronger, more stable
Learning behavior	Task engagement	Hesitant, avoidant	Active, confident
	Dependency on aids	High (fingers, drawings)	Low (independent)
Mathematics anxiety	Fear of making mistakes	Moderate–high	Low–moderate
	Nervousness toward symbols	High	Low
	Willingness to attempt tasks	Low	Increased

Table 1 summarizes the overall outcomes of the CPA-based intervention by comparing H's performance before and after the program. The results indicate a substantial improvement in mathematical accuracy, increasing from 41.6% at pre-test to 83% at post-test, exceeding the predetermined success criterion. Qualitatively, H showed a clear shift in calculation strategies from reliance on finger counting and tally marks to more independent mental calculation, suggesting improved conceptual understanding of numerical operations.

In addition to cognitive gains, notable changes were observed in behavioral and affective domains. Prior to the intervention, H demonstrated avoidance behavior, high dependency on concrete aids, and anxiety when facing mathematical symbols. Following the CPA intervention, engagement increased, avoidance behavior decreased, and anxiety toward symbols was markedly reduced. These findings indicate that the CPA method not only enhanced numerical accuracy but also strengthened representational connections and improved emotional readiness for symbolic mathematical tasks

The counseling session began with a psychological assessment session using the client centered case consultation model as described by Henderson & Thompson (2016) in (Tharinger, et al., 2022), which focused on helping the consultant understand the child's condition and determine the most appropriate intervention steps (Leeming et al., 2022). In this session, the examiner comprehensively explained H's cognitive and affective profile to the parents, including visual-spatial strengths and challenges in processing mathematical symbols. The counseling session lasted 40–60 minutes and included an exploration of the child's symptoms, clarification of parental expectations, explanation of the psychological assessment results, discussion of the diagnosis, and establishment of intervention goals. The final phase of counseling focused on ensuring the parents' understanding of the Concrete Pictorial-based intervention plan.

The intervention program was designed based on assessment findings indicating that H had difficulty understanding quantities, performing two-digit addition and subtraction, translating problems into symbols, and exhibiting anxiety when dealing with numbers. These problems were related to weak mental representations of numbers, a reliance on concrete strategies, and inconsistencies in connecting real objects to images and symbols. The CPA intervention was chosen because it aligns with the concrete operational stage of development and has been proven effective for elementary school students with math difficulties.

The primary focus of the intervention was to improve understanding of numerical concepts, strengthen representational connections, and reduce math anxiety. The general goal of the program was to help H complete two-digit operations more meaningfully, while specific goals included connecting concrete objects, images, and symbols; improving calculation accuracy; reducing avoidance behavior; and building self-confidence. Short-term goals emphasized mastery of concrete manipulation and visual representation; medium-term goals focused on symbol recognition; and long-term goals included the ability to solve two-digit problems without the aid of media and achieve at least 80% accuracy.



The intervention was conducted individually through four core sessions spanning approximately 12 sessions, including a pre-test and post-test. Each session reflected the stages of CPA: concrete block manipulation to understand union and remainder; drawing visual representations; transitioning to basic symbols; and advanced symbolic practice without the aid of pictures. The intern provided verbal scaffolding, positive reinforcement, and a stress-free learning environment to reduce H's anxiety. Media such as colored blocks, number cards, and worksheets were used to ensure consistency of representation. The intervention process was observed through behavioral observation sheets and a cognitive rubric.

Evaluation of program success was conducted through comparison of pre-test and post-test scores, process observations, and a math anxiety checklist. Success was assessed based on an increase in calculation accuracy to  $\geq 80\%$ , increased engagement, and decreased avoidance behavior. After the four core sessions, a termination session focused on reflecting on the intervention results, consolidating calculation strategies, and developing further recommendations. A follow-up session was conducted 1 week later to assess retention of symbolic skills and to ensure that H could count two digits independently without assistive devices.

## Counseling Implementation Results

**Table 2.**  
**Summary of Counseling Implementation Outcomes**

Aspect Evaluated	Indicator	Outcome
Counseling engagement	Parent participation	Active, cooperative, and attentive
	Counselor–parent rapport	Warm and supportive
Problem clarification	Parental understanding of child's difficulties	Clear and accurate
	Identification of core issues	Math-specific difficulties and math anxiety
Assessment comprehension	Understanding of psychological assessment results	Well understood and accepted

	Recognition of child's strengths and weaknesses	Accurate acknowledgment
Emotional response	Parental emotional reaction	Relief and increased hope
Intervention readiness	Acceptance of CPA intervention plan	Fully agreed
	Willingness to support intervention at home	High
Home support planning	Adoption of recommended strategies	Positive reinforcement, concrete aids, stress-free environment

Table 2 summarizes the outcomes of the counseling implementation session. The results indicate that the counseling process was effective in establishing a supportive counselor–parent relationship and enhancing parental understanding of H's mathematical difficulties. Mrs. H demonstrated active participation, accurately comprehended the psychological assessment results, and acknowledged both H's strengths and challenges. Importantly, the counseling session contributed to a positive emotional shift, reflected in reduced parental concern and increased optimism regarding H's learning potential.

Furthermore, the counseling session successfully facilitated readiness for intervention. Mrs. H expressed full acceptance of the CPA-based intervention plan and demonstrated high willingness to implement recommended home support strategies, including the use of concrete learning aids, step-by-step practice, and positive reinforcement. These outcomes indicate that the counseling process functioned as an effective foundation for the subsequent intervention phase

The counseling session began with a pre-counseling phase, where the intern established a warm atmosphere through greetings and casual conversation with Mrs. H, the primary caregiver. Initial conversations about H's study habits and responses to math lessons helped establish a comfortable working relationship. Mrs. H appeared open and cooperative, responding positively and showing full attention to the counseling process. Once a conducive atmosphere was established, the intern reviewed H's complaints. Based on observations and informal assessments, H demonstrated strong literacy skills but struggled with numeracy, particularly with two-digit addition and subtraction and the use of mathematical symbols. Characteristics of math anxiety were evident through negative comments about math, repetitive finger usage, and long pauses before working on problems. Mrs. H confirmed this



description and explained that H often complained when asked to do math at home.

The intern then presented the results of the psychological assessment in detail, covering H's academic abilities, calculation strategies, and emotional dynamics when dealing with numerical tasks. This explanation was well received by Mrs. H, who acknowledged that H tends to be slow at calculations and becomes easily anxious when faced with numbers, despite being quite confident in reading and writing tasks. The problem analysis phase revealed that H's challenges were specific, related to his understanding of numerical concepts, instability in two-digit operations, and math anxiety that increased his cognitive load.

In the prognosis phase, the intern explained that H's math abilities could improve significantly with concrete, gradual, and consistent intervention. This explanation brought a sense of relief to Mrs. H, who expressed hope that H would develop a fearless understanding of mathematics. The intern then outlined the intervention program, the Concrete Pictorial Abstract (CPA) method, which includes manipulating concrete objects, drawing visual representations, transitioning to basic symbols, and ultimately, the independent use of mathematical symbols.

During the counseling session, the intern recommended several home support strategies, such as providing step-by-step math practice, using concrete objects when H seemed hesitant, linking counting to daily activities, and providing positive reinforcement to boost confidence. A calm and stress-free learning environment was also emphasized to reduce math anxiety. Overall, the counseling session was effective, with Mrs. H's active participation and cooperation, thereby opening the door to continued interventions that could support H's improvement in math skills and reduce her math anxiety.

## **Intervention Results**

### **1. Pre-test Results**

The pre-test was conducted to assess H's initial abilities in two-digit mathematical operations, including addition, subtraction, mixed operations, and translating word problems into mathematical form. The pre-test was also used to determine H's spontaneous cognitive strategies and emotional responses to math tasks before receiving the CPA intervention.

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**Table 3. The following table shows the answer to H.**

No	Question	Answer	Key H	B/S
1	$27 + 16$	43	43	√
2	$34 + 28$	62	60	X
3	$56 + 17$	73	73	√
4	$63 - 25$	38	40	X
5	$72 - 38$	34	33	X
6	$48 - 19$	29	29	√
7	$91 - 46$	45	50	X
8	$37 + 29 - 15$	51	47	X
9	$54 - 27 + 18$	45	45	√
10	$35 + 18$	53	53	√
11	$72 - 26$	46	40	X
12	$6 \times 3$	18	19	X

Total correct: 5 out of 12 Accuracy: 41.6%

The preliminary test in Table 3 shows that H's initial ability in two-digit mathematical operations is still unstable. H appeared hesitant when working on problems, using concrete strategies such as counting on his fingers and making small scribbles, and showed signs of anxiety, especially in two-digit subtraction and mixed operations. The preliminary test accuracy was only 41.6%, and most errors occurred in two-digit subtraction and word problems that require symbolic understanding. The math anxiety scale confirmed that H was in the moderate-high anxiety category, especially in items related to fear of making mistakes and nervousness when seeing number symbols. This condition suggests that H requires a gradual approach that reduces cognitive load and helps build conceptual understanding slowly.

In the Concrete session, H responded positively to the use of Dienes blocks. He appeared calmer and more enthusiastic, and grasped the concepts of combination and subtraction more easily through the manipulation of physical objects. Although subtraction still caused confusion, the use of real objects reduced anxiety and helped H focus. The immediate success of seeing the results of the manipulatives provided a sense of security and increased H's confidence. H's performance in this session indicated that concrete manipulatives were effective as a foundation for understanding basic operations.

Entering the Pictorial session, H was able to transform concrete understanding into pictorial form. He carefully drew a representation of the sum, double-checked the results, and began to explain his thinking steps verbally. The visual representation helped H re-envision the concrete process he had experienced, leading to fewer errors and a stronger conceptual understanding. The anxiety that emerged in the symbol stage began to subside as the images served as a cognitive buffer, stabilizing his thinking.

In the Early Abstract session, H began working with two-digit math symbols. Although he again showed signs of nervousness at the beginning of the session, he was able to link the symbols to previous concrete and visual experiences. He used verbal strategies such as mentally restating the steps and showed improvement in two-digit addition, although subtraction remained slow. His ability to connect images to symbols strengthened, indicating that the concept was beginning to be internalized.

In the Advanced Abstract session, H was able to complete two-digit addition and subtraction without the aid of images or concrete objects. He began to apply simple mental strategies, such as adding tens first, and showed increased speed and accuracy. He still had difficulty with subtraction, but the intensity was much lower than in the initial session. H appeared more confident, did not exhibit the same anxious gestures as before, and was able to solve mixed problems after minimal guidance regarding the order of work. These positive behavioral changes indicate that the CPA intervention not only improved numeracy skills but also significantly reduced math anxiety.

The intervention results showed steady and consistent development from the concrete stage to symbolic abstraction. H maintained this understanding until the final session and even demonstrated readiness for the post-test evaluation. CPA proved effective as a step-by-step approach that helped H build conceptual understanding, reduce cognitive load, and increase confidence in mathematics.

## **2. Post Test Results**

During the post-test, changes in H's behavior were clearly visible. He sat in a stable position, no longer delaying his start as he had in the pre-test. When given the problem paper, he immediately read the first line and began writing his answers with a more consistent rhythm. There were no signs of anxiety, such as sighing repeatedly, fiddling with his pencil, or staring at the problem without moving. H handled addition operations calmly and fairly quickly. With subtraction, he appeared more cautious, but was able to

maintain focus without appearing as flustered as before. He no longer relied on his fingers or small scribbles; some calculations were performed mentally.

With mixed operations and word problems, H paused briefly to think, but not out of confusion, but rather to process the information in a structured manner. Throughout the work, H appeared confident. He no longer sought confirmation from the facilitator through eye contact. When completing the final problem, he smiled slightly, indicating that his previous learning experience had given him a sense of control and competence.

**Table 4. Post-Test Results**

No	Question	Answer	Key H	B/S
1	$28 + 17$	45	45	√
2	$47 + 26$	73	73	√
3	$58 + 19$	77	76	X
4	$69 - 24$	45	45	√
5	$83 - 37$	46	46	√
6	$52 - 18$	34	34	√
7	$94 - 45$	49	50	X
8	$42 + 38 - 17$	63	63	√
9	$61 - 26 + 20$	55	55	√
10	$46 + 27$	73	73	√
11	$90 - 34$	56	56	√
12	$8 \times 4$	32	32	√

Score: 10/12

Accuracy: 83%

The post-test results in Table 4 show an improvement in the subjects' numerical abilities after the Concrete Pictorial Abstract (CPA) intervention. Of the 12 questions covering two-digit addition and subtraction, mixed operations, and one simple multiplication problem, the subjects successfully answered 10 questions correctly with an accuracy rate of 83%. Performance on two-digit addition appeared stable and consistent, while one procedural error was found in two-digit subtraction. The subjects were also able to solve mixed operations well and correctly answer simple multiplication problems, although this was not the primary focus of the intervention. Overall, these data demonstrate a more stable development of conceptual understanding and use of mental

strategies at the abstract stage, and are presented descriptively as indicators of changes in learning performance within the context of the case study.

**Table 5. Post-Test Math Anxiety Scale Results Table**

No	Statement	Answer H	Score	Interpretation
1	I feel afraid of making mistakes when calculating	Sometimes	2	Anxiety is reduced
2	I enjoy learning with real objects like blocks	Strongly Agree	4	Positive preferences for CPA learning
3	I get nervous when I see number symbols.	Disagree	1	The symbols are no longer scary
4	I'm more daring to try math problems now.	Agree	3	Self-confidence increases
5	I enjoy learning math more than before.	Agree	3	Mathematical attitudes improve

Total Post-Test Score:  $2 + 4 + 1 + 3 + 3 = 13$

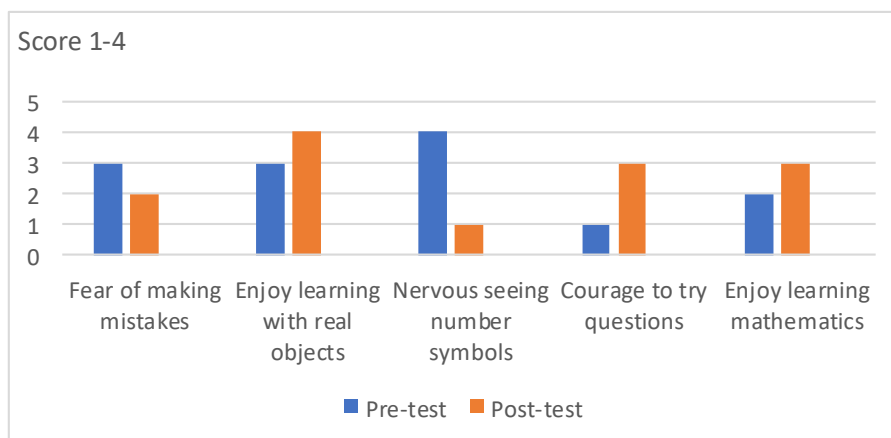
Although the total pre-test and post-test scores were numerically the same, the composition of the post-test scores changed drastically, indicating: decreased anxiety (negative items decreased), increased interest and courage (positive items increased). Category: Low-Moderate Math Anxiety, with an increase in positive attitudes.

The results of the math anxiety measurement in Table 5 indicate positive changes in the subjects' affective aspects after the CPA intervention. Although the total post-test score was 13, numerically the same as the initial score, the composition of the responses showed a significant shift. Scores on negative statements, such as fear of making mistakes and nervousness when seeing number symbols, decreased, while scores on positive statements, including courage to try math problems, enjoyment of learning with concrete media, and positive attitudes toward mathematics, increased. Subjects reported no longer feeling nervous around mathematical symbols and showed a strong preference for concrete object-based learning. Overall, these results indicate a decrease in math anxiety to the low-moderate category accompanied by an increase in positive attitudes and confidence in learning mathematics.

**Table 6. Comparison of Pre-Post Math Anxiety Change Scales**

Statement	Pre-Test	Post-Test	Change
Fear of making mistakes	3	2	↓ decreases
Enjoys learning with real objects	3	4	↑ increase
Nervousness when looking at number symbols	4	1	↓ greatly decreased
Courage to try problems	1	3	↑ increased strongly
Enjoys learning math	2	3	↑ increase

The data in Table 6 show that H's math anxiety decreased significantly after the implementation of CPA. Anxiety scores, previously dominated by negative items ("fear of making mistakes," "nervous about symbols"), decreased significantly, while positive items ("dare to try," "enjoy learning") increased. These changes are consistent with H's behavior during the intervention and post-test, which showed an increased sense of control, focus, and motivation to learn.



**Figure 1. Pre-Post Comparison Chart of Math Anxiety Scales**

The graph in Figure 1 above shows the change in H's math anxiety levels before and after the CPA intervention based on the five statements in the Math Anxiety Checklist. Overall, the graph shows a trend of decreasing anxiety and increasing comfort in learning mathematics after four CPA intervention sessions.



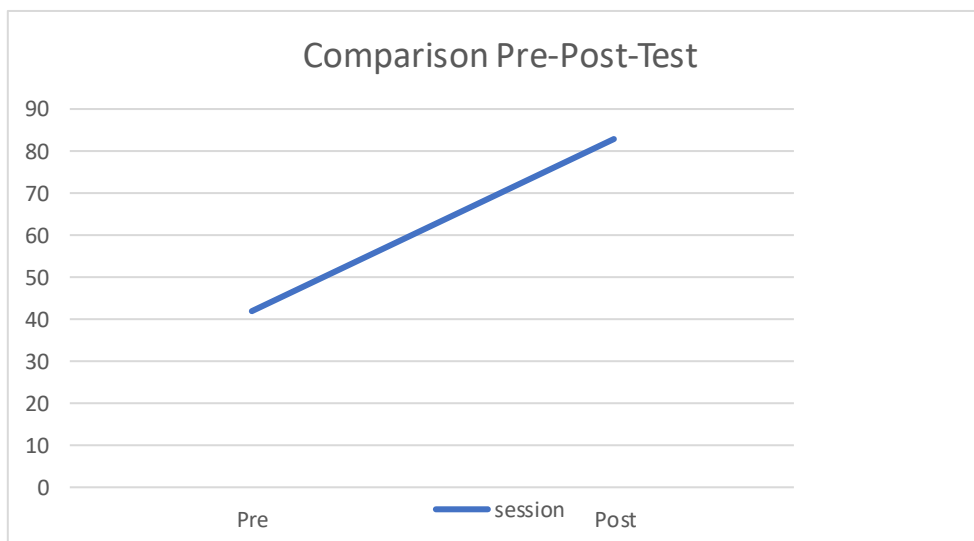
For the statement "Fear of making mistakes when calculating," H's score decreased from 3 to 2, indicating that H felt more secure and less stressed when working on problems. The most drastic decrease was observed in the statement "Nervous about number symbols," from 4 (strongly agree) to 1 (disagree). This indicates that H no longer considered number symbols to be an anxiety trigger and had become more familiar with mathematical notation after progressing through the abstract stage.

On the other hand, the positive aspects of learning mathematics increased. For the statement "Enjoys learning with real objects like blocks," the score increased from 3 to 4, indicating that concrete methods significantly contribute to H's comfort. Meanwhile, "Courage to try math problems" increased significantly from 1 to 3, indicating increased self-confidence. The final statement, "Enjoys learning math," also increased from 2 to 3, indicating that H is beginning to build positive associations with counting activities. Overall, the graph shows that the CPA intervention not only improved H's cognitive abilities but also directly reduced math anxiety and increased learning motivation.

The post-test results showed a striking improvement compared to his baseline performance before the intervention. At this stage, H was able to complete all math operations without resorting to manipulatives or visual aids. His calculation skills appeared significantly more independent; he solved two-digit addition problems with a more steady rhythm and no longer relied on his fingers or small scribbles as compensatory strategies. Significant changes were also seen in his two-digit subtraction skills. Whereas H had previously appeared hesitant, paused for long periods, and frequently made errors during the borrowing process, he was now able to complete these operations with greater confidence and accuracy. The small errors that did occur were mechanical, such as misspelling numbers, rather than stemming from a lack of conceptual understanding. This indicates that his conceptual structure had become much more mature than at the pre-test.

In addition to improved accuracy, H's mental calculation strategies have also developed. He began breaking numbers into tens and ones spontaneously, indicating that he wasn't simply memorizing steps but truly understanding the relationships between numbers. H no longer appeared confused when choosing a strategy, and his calculation process felt more efficient and less choppy than before. Emotionally, positive changes were evident. H, who initially showed signs of anxiety such as procrastination, fidgeting, and negative comments about math, now appeared calmer and more focused while

working on problems. He no longer exhibited avoidance behavior or repeated requests for reassurance, indicating a decrease in his math anxiety. On several occasions, he even smiled when solving problems correctly, demonstrating a growing confidence in his own abilities.



**Figure 2. Progress Graph Comparison of Pre-Test and Post-Test**

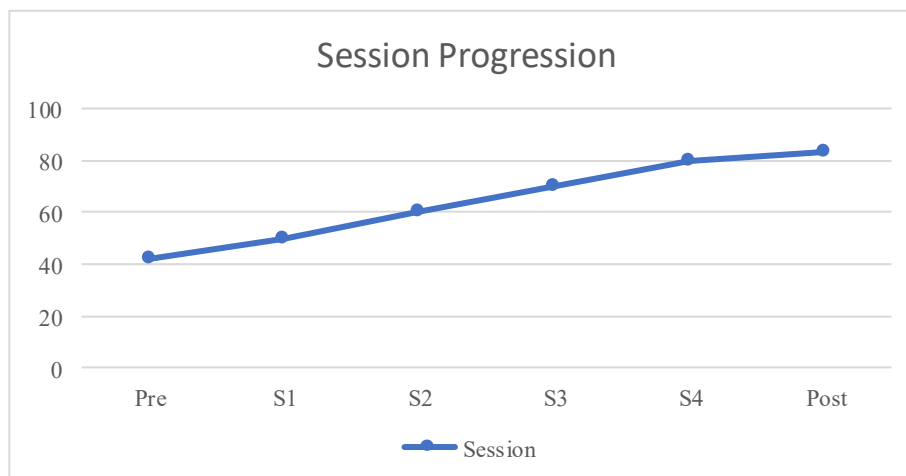
The Comparison Chart in Figure 2 above Pre-Post-Test shows a significant improvement in H's two-digit arithmetic ability after the CPA intervention. Initially, H's accuracy in the pre-test was 42%, indicating that H still had difficulties, especially in two-digit subtraction and mixed operations. After going through four stages of intervention (Concrete, Pictorial, Early Abstract, and Advanced Abstract), H's accuracy increased to 83% in the post-test.

This improvement reflects H's better understanding of the basic concepts of mathematical operations and the beginning of using simple mental strategies for calculation. This nearly twofold increase also indicates that the CPA method was effective in helping H build a gradual understanding from concrete objects to mathematical symbols.

**Table 7. Progress Per Session**

Stage	Accuracy
Pre-test	42%
Session 1	50%
Session 2	60%
Session 3	70%
Session 4	78%
Post-test	83%

The results of the math anxiety measurement in Table 7 indicate a change in the subjects' affective response patterns after the Concrete Pictorial Abstract (CPA) intervention. Although the total post-test score was 13, numerically equivalent to the pre-test score, the distribution of scores showed a significant shift. Responses to negative statements, such as fear of making mistakes and nervousness when seeing mathematical symbols, decreased, while responses to positive statements, including enjoyment of learning using concrete objects, courage to try problems, and positive attitudes toward mathematics learning, increased. Subjects explicitly stated that they no longer felt nervous when faced with number symbols and showed a strong preference for concrete-based learning. Descriptively, these findings indicate a decrease in the intensity of math anxiety to the low–moderate category, accompanied by an increase in positive attitudes and confidence in calculating activities, which aligns with the changes in learning behavior observed during and after the intervention.



**Figure 3. Progress Graph Per Session Progress Graph Per Session**

The Progress Chart per Session in Figure 3 shows H's gradual improvement at each stage of the CPA intervention. In Session 1 (Concrete), H's accuracy increased from 42% to 50%, indicating that manipulating real objects helped H understand the basic concepts of numbers and remainders.

In Session 2 (Pictorial), the increase continued to 60%, indicating that H was able to transfer concrete understanding to pictorial representations with relative stability. Entering Session 3 (Early Abstract), accuracy increased again to 70%, indicating that H was beginning to understand the symbols "+," "-", and "=" as abstract representations of pictures.

In Session 4 (Advanced Abstract), accuracy increased to 78% when H was asked to solve purely symbolic problems without the aid of pictures or manipulatives. Finally, after all sessions were completed, accuracy on the post-test reached 83%, indicating stable and consistent improvement in H's numeracy skills. This chart depicts a positive linear progression, in line with the gradual principles of the CPA model.

### 3. Follow-up

A follow-up session was conducted in the 12th session, approximately one week after the termination session. The purpose of this phase was to assess H's retention of two-digit counting skills without the aid of concrete tools or pictures, and to observe the stability of his self-confidence and counting strategies. In this phase, H was asked to

work on five two-digit problems without aids, with a difficulty level equivalent to the advanced abstract stage. H demonstrated stable performance, accurately answering 4 out of 5 problems (80% accuracy), meeting the retention target set in the intervention design.

During the follow-up, H no longer exhibited the dominant hesitant behaviors seen in the pre-test. He was able to begin working on problems without delay, did not rely on finger counting, and used simple decomposing strategies, such as breaking numbers into tens and ones before counting. He also appeared emotionally calmer and displayed a confident expression when stating his answers. From an affective perspective, H's math anxiety appeared to decrease consistently. He did not complain or show tension when given problems, even saying, "I'll try first, Mom," several times, indicating increased motivation and courage to face math tasks.

Follow-up results showed that H's symbolic abilities not only improved during the intervention, but also persisted after the intervention was completed, indicating that the CPA stage had helped build a more stable conceptual understanding.

**Table 8. Follow-Up Results on Two-Digit Arithmetic Performance and Strategy Retention**

No	Question <i>Follow-Up</i>	Answer H	True/False	Time (seconds)	Notes
1	$26 + 17 =$	43	√	18 seconds	H counts without fingers; uses the tens + ones strategy.
2	$54 - 28 =$	26	√	25 seconds	Hesitates slightly before answering, but is able to correct himself.
3	$39 + 25 =$	64	√	20 seconds	Quick response; appears confident.
4	$72 - 19 =$	63	√	22 seconds	Using decomposition: "Seventy-two minus ten is 62, minus nine is 63."
5	$48 - 27 =$	25	X	30 seconds	Error on loan; H was confused between 8-7 and 14-7.

**Table 9. Summary of Counseling Implementation Outcomes**

Components	Results
Number of Questions	5
True	4
False	1
<b>Follow Up Accuracy</b>	<b>80%</b>
Verage Time	±23 seconds/question
Conclusion	Double digit ability retention is good, mental strategy is starting to stabilize.

The follow-up results, conducted one week after the termination session, shown in Table 9, indicate that the subject's two-digit counting ability was maintained stably. Based on the data in the follow-up table, the subject was able to answer 4 out of 5 questions correctly, with an accuracy rate of 80%, in line with the retention target set in the intervention design. During the task, the subject no longer displayed dominant hesitation behavior, did not rely on finger counting, and began to use simple mental strategies such as decomposing (breaking numbers into tens and ones). The errors that still occurred were procedural in nature, particularly in subtraction with borrowing, and did not reflect a complete conceptual failure. Affectively, the subject appeared calmer and more confident, and demonstrated the courage to attempt to solve problems without pressure. Descriptively, these findings indicate that the symbolic understanding and counting strategies developed through the CPA stages not only improved during the intervention but also persisted after the intervention was completed, indicating the formation of a relatively stable conceptual understanding.

The CPA intervention demonstrated strong effectiveness in improving H's numerical abilities, and the developmental patterns that emerged during the intervention align with Bruner's (1966) representational framework. Based on this theory, ideal mathematics learning proceeds through three stages: enactive (or concrete) representation, iconic (or visual) representation, and symbolic representation. H demonstrated development consistent with this representational sequence; he gradually moved from manipulating concrete objects to images as intermediate representations, ultimately working with abstract symbols without external assistance.

H's increased accuracy demonstrates that each stage of CPA plays a crucial role in developing a comprehensive mathematical understanding. In the concrete stage, the use



of blocks helped reduce working memory load, allowing H to understand operations directly and concretely. As he progressed to the pictorial stage, his self-created visualizations strengthened his cognitive structures regarding addition and subtraction. In the early abstract stage, the relationship between images and symbols began to be internalized, enabling H to read and process mathematical operations more fluently. The advanced abstract stage then provided space for H to develop mental calculation, the ability to calculate independently through internal operations without the aid of manipulative tools.

This intervention also impacted affective aspects, particularly a reduction in math anxiety. These changes are consistent with the findings of Ashcraft and Moore (Ashcraft & Moore, 2009), who explained that success achieved in a supportive learning environment can reduce math anxiety. During the intervention, H experienced gradual and structured success, which made him feel more capable and less threatened by mathematical symbols. This decrease in anxiety was evident in more relaxed physical expressions, higher engagement, and reduced avoidance behavior during the sessions.

Follow-up results confirmed these findings. An 80% retention rate indicated that H not only understood operations during the intervention sessions but also retained these abilities long after the intervention ended. This retention was particularly evident in his ability to combine and separate quantities, use mental strategies such as decomposing, and interpret mathematical symbols independently. These findings align with Bruner's idea that concrete and visual representations help form more stable cognitive structures, allowing concepts to be retained even after aids are removed.

From an emotional perspective, reduced anxiety appears to contribute to the sustainability of learning outcomes. In a more positive environment and with accumulated success, H became calmer and more confident when faced with math problems, as predicted by math anxiety theory. Thus, the CPA intervention not only improved H's numerical abilities during implementation but also laid a cognitive and emotional foundation that supported his short-term learning independence.

## CONCLUSION

This study aimed to describe the cognitive and affective profile of a child with mathematics learning difficulties and to analyze changes in conceptual understanding following the implementation of the Concrete–Pictorial–Abstract (CPA) approach. The findings indicate

that the child's mathematical difficulties were associated with limitations in symbol processing, unstable understanding of two-digit operations, and elevated mathematics anxiety, despite having adequate intellectual potential. Through the staged CPA intervention, the child demonstrated gradual improvements in numerical understanding, moving from reliance on concrete representations to the use of basic mental calculation strategies at the abstract level. Descriptive results showed increased accuracy in two-digit arithmetic, reduced dependence on concrete aids, and improved stability in problem-solving strategies. In addition, positive affective changes were observed, including reduced mathematics anxiety, increased confidence, and greater willingness to engage in mathematical tasks. Follow-up results further indicated that the conceptual understanding developed through the CPA stages was retained after the intervention period, suggesting the formation of a more stable and sustainable numerical representation. Overall, this study highlights the importance of aligning instructional approaches with children's cognitive developmental stages and demonstrates the potential of the CPA approach to support both cognitive and affective aspects of mathematics learning in children with specific learning difficulties.

## REFERENCES

- Ashcraft, M. H., & Moore, A. M. (2009). Mathematics Anxiety and the Affective Drop in Performance. *Journal of Psychoeducational Assessment*, 27(3), 197–205. <https://doi.org/10.1177/0734282908330580>
- Barbieri, C. A., Rodrigues, J., Dyson, N., & Jordan, N. C. (2020). Improving fraction understanding in sixth graders with mathematics difficulties: Effects of a number line approach combined with cognitive learning strategies. *Journal of Educational Psychology*, 112(3), 628–648. <https://doi.org/10.1037/edu0000384>
- Bruner, J. S. (1966). *Toward a Theory of Instruction* (Vol. 59). Harvard University Press.
- Chen, W. (2024). Problem-Solving Skills, Memory Power, and Early Childhood Mathematics: Understanding the Significance of the Early Childhood Mathematics in an Individual's Life. *Journal of the Knowledge Economy*, 16(1), 1–25. <https://doi.org/10.1007/s13132-023-01557-6>
- Grigorenko, E. L., Compton, D. L., Fuchs, L. S., Wagner, R. K., Willcutt, E. G., & Fletcher, J. M. (2020). Understanding, educating, and supporting children with specific learning disabilities: 50 years of science and practice. *American Psychologist*, 75(1), 37–51. <https://doi.org/10.1037/amp0000452>

- Hasibuan, A. T., & Prastowo, A. (2019). Konsep Pendidikan Abad 21: Kepemimpinan dan Pengembangan Sumber Daya Manusia Sd/Mi. *MAGISTRA: Media Pengembangan Ilmu Pendidikan Dasar Dan Keislaman*, 10(1). <https://doi.org/10.31942/mgs.v10i1.2714>
- Hasibuan, A. T., Sianipar, M. R., Ramdhani, A. D., Putri, F. W., & Ritonga, N. Z. (2022). Konsep dan Karakteristik Penelitian Kualitatif Serta Perbedaannya Dengan Penelitian Kuantitatif. *Jurnal Pendidikan Tambusai*, 6(2), 8686–8692.
- Hasibuan, A. T., Simatupang, W. W., Rudini, R., & Ani, S. (2023). Implementasi Sistem Pendidikan Terbaik Dunia di Jenjang Anak Usia Dasar Telaah Sistem Pendidikan Finlandia. *JURNAL PEMBELAJARAN DAN MATEMATIKA SIGMA (JPMS)*, 9(1). <https://doi.org/10.36987/jpms.v9i1.4383>
- Holmes, J., Guy, J., Kievit, R. A., Bryant, A., Mareva, S., & Gathercole, S. E. (2021). Cognitive dimensions of learning in children with problems in attention, learning, and memory. *Journal of Educational Psychology*, 113(7), 1454–1480. <https://doi.org/10.1037/edu0000644>
- Kroesbergen, E. H., Huijsmans, M. D. E., & Friso-van den Bos, I. (2023). A Meta-Analysis on the Differences in Mathematical and Cognitive Skills Between Individuals With and Without Mathematical Learning Disabilities. *Review of Educational Research*, 93(5), 718–755. <https://doi.org/10.3102/00346543221132773>
- Leeming, D., Marshall, J., & Hinsliff, S. (2022). Self-conscious emotions and breastfeeding support: A focused synthesis of UK qualitative research. *Maternal & Child Nutrition*, 18(1). <https://doi.org/10.1111/mcn.13270>
- Lein, A. E., Jitendra, A. K., & Harwell, M. R. (2020). Effectiveness of mathematical word problem solving interventions for students with learning disabilities and/or mathematics difficulties: A meta-analysis. *Journal of Educational Psychology*, 112(7), 1388–1408. <https://doi.org/10.1037/edu0000453>
- Marinda, L. (2020). Teori Perkembangan Kognitif Jean Piaget Dan Problematikanya Pada Anak Usia Sekolah Dasar. *An-Nisa Jurnal Kajian Perempuan Dan Keislaman*, 13(1), 116–152. <https://doi.org/10.35719/annisa.v13i1.26>
- Santrock, J. W. (2011). *Life-Span Development*. McGraw Hill.
- Tharinger, D. J., Rudin, D. I., Frackowiak, M., & Finn, S. E. (2022). *Therapeutic Assessment with Children: Enhancing Parental Empathy Through Psychological Assessment* (1 ed.). Routledge. <https://doi.org/10.4324/9781003000174>

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Zhou, D., Du, X., Hau, K.-T., Luo, H., Feng, P., & Liu, J. (2020). Teacher-student relationship and mathematical problem-solving ability: Mediating roles of self-efficacy and mathematical anxiety. *Educational Psychology*, 40(4), 473–489. <https://doi.org/10.1080/01443410.2019.1696947>